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風險值方法實證研究－以一壽險公司為例

AN EMPIRICAL TEST ABOUT THE METHODOLOGY OF VALUE-AT-RISK ON AN LIFE INSURANCE COMPANY

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Keywords

Market Risk, Reduced-formed, Univariate method, Value-at-Risk (VaR)

Abstract

Value-at-Risk (VaR), nowadays, is the most widely adopted risk management method for measuring market risk in financial institutions, like banks, securities companies, and insurance companies etc. Although this measure is so widespread, it has some setbacks. In recent year, trading activities in financial institutions have grown substantially and became progressively more diverse and complex. In this situation, the complicate structural models were not able to outperform a simple univariate model in terms of accuracy and forecasting ability in 99^{th} percentile. Univariate models, therefore, are at least a useful complement to large structural models and might even be sufficient for forecasting VaR. This paper is the first article that shows univariate method with historical data from a life insurance compnay in Taiwan and provides a comparison of the performance between the univariate one and the models actually in use within firm.

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Acknowledgements

After studying in NCCU for six years, I finally have to leave here. This is really a turning point to me right now. In the past six years, a lot of things happened,

Chapter 1: Introduction

In recent year, the trading position at insurance company in Taiwan have increased and become more complex. According to Table 1, Taiwan Insurance Institute’s data demonstrates that the total amount of capital invested has grown 84% from 2008 to 2013.

In order to manage market risks, major trading institutions have developed large-scale risk measurement models. While approaches might be different, all such models measures and aggregate market risks in current positions at a highly detailed level. The models employ a standard risk metric, Value-at-Risk (VaR), which is a lower tail percentile for the distribution of profit and loss (P&L).

VaR models have been adopted for determining capital requirement under Solvency II. Article 101 of the Solvency II Directive states, "The Solvency Capital Requirement (SCR) shall correspond to the Value-at-Risk (VaR) of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period." Spurred by these developments, VaR has become a standard measure of market risk that is increasingly used by financial firms and non-financial firms.

Table 1. Total Amount of Capital Invested of Taiwan Insurance Companies

Year	Total Amount of Capital Invested (million, NTD)
2008	7,981,732
2009	9,262,558
2010	10,486,298
2011	11,468,150
2012	12,758,605
2013	14,677,656

Source: Taiwan Insurance Institute, retrieved May 18, 2014

The general acceptance and use of large-scale VaR models has a substantial literature including statistical descriptions of VaR and examinations of different modelling issues and approaches. However, because of the proprietary nature, there has been little empirical study of risk models actually in use. Berkowitz and O'Brien (2001) provided the first direct evidence on the performance of bank VaR models. They analyse the distribution of historical trading P&L and the daily performance of VaR estimates of 6 large U.S. banks from January 1998 to March 2000.

In this paper, we follow Berkowitz and O'Brien's method and use the P&L data of one insurance company in Taiwan to see the performance of reduced form model compared to internal structural model.

The rest of the article is organized as follows. Chapter 2 has an overall review for VaR method and motivation of this research. Chapter 3 presents the research methods adopted by this paper. Chapter 4 and Chapter 5 provide the results and conclusions. The final chapter lists some suggestions for future research.

Chapter 2: Literature Review

In this chapter, we will go through the development of VaR and the regulatory of it as well. In this paper, however, we mainly pay attention to insurance company instead of bank. Hence, we will also cover review of economic capital. In the end, we will talk about the limitations of internal structural model following with the reduced form model method- the aim of this paper.

2.1 THE RISE OF VALUE-AT-RISK

In Jorion (2001), he wrote that Value-at-Risk (VaR) can be given the following intuitive definition: VaR summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence.

VaR, nowadays, is a standard measure for financial market risk that is popularly used by other financial and even non-financial firms as well. But, in fact, many financial firms lacked an independent risk management function in 1990. Holton et al. (2002) went through the early development of VaR. At that time, the terminology “risk management” was not new, but it was used to describe techniques for property and casualty contingencies. Those techniques, added traditional insurance, were collectively referred to as risk management. As time passing by, derivative dealers promoted “risk management” as the use of derivatives to hedge or customize market-risk exposures.

In the summer, 1992, Paul Volker, chairman of the Group of 30[1], approached Dennis Weatherstone, chairman of JP Morgan, and asked him to lead a research of derivatives industry practices. Weatherstone and his team produced a 68-pages report, which published by the Group of 30 in July 1993, entitled *Derivatives: Practices and Principles*, which has been known as the *G-30 Report* right now. With concerns to the market risks faced by derivatives dealers, the report recommended that portfolio

[1] Found in 1978, the Group of 30 is a non-profit organization of senior executives, regulators and academics. Through meetings and publications, it seeks to deepen understanding of international economic and financial issues.

should be marked-to-market daily, and that risk be assessed with both VaR and stress testing. It recommended that end-users of derivatives implement similar practices as appropriate for their own needs.

Despite the *G-30 Report* focused on derivative products, most of its recommendations were applicable to the risks associated with other traded products. Therefore, the report defined the new risk management of the 1990's. The report also might be the first published document to bring out the word "Value-at-Risk."

In late 1980's, JP Morgan developed a firm-wide VaR system[2]. This modeled several hundred of risk factors. A covariance matrix was updated quarterly from historical data. Each day, trading units would be reported by e-mail their positions' deltas with respect to each of risk factors. These were aggregated to express the combined portfolio's value as a linear polynomial of the risk factors. Through this process, the standard deviation of portfolio was calculated. Various VaR metrics were employed. One of these was one-day 95% VaR, which was computed using an assumption that the portfolio's value was normally distributed.

With this VaR measure, JP Morgan replaced a cumbersome system of notional market risk limits with an easy system of VaR limits. Commencing in 1990, VaR numbers were combined with P&Ls in a report for each day's 4:15 pm Treasury meeting in New York. Those reports, with comments of the Treasury group, were forwarded to Chairman Weatherstone. Afterward, Till Guldimann, one of the architects of the new VaR measure in JP Morgan, developed a service named RiskMetrics. It contains a detailed technical document as well as a covariance matrix for several hundred of important risk factors, which will be updated daily. While RiskMerics was not a breakthrough technique and its method is even less sophisticated than most of other methods at the time, its contribution was that it promoted VaR to the public.

2.2 REGULATORY APPROVAL OF PROPRIETARY VAR MEASURES

In 1993, following the fiasco of its joint initiative, which aimed to harmonize the market risk capital requirements of banks and securities firms in worldwide, with

[2] See Guildimann (2000)

IOSCO[3], the Basle committee released amendments to its 1988 accord. These amendments included a document proposing Minimum Capital Requirements for market risk of banks. The proposal generally accord with Europe's CAD[4]. Banks would be required to identify a trading book and prepare capital for trading book market risks and organization-wide foreign exchange exposures. Capital charges for the trading book would be based upon a building block VaR measure, which entails separate "general risk" and "specific risk" computation, roughly consistent with a 10-day 95% VaR metrics. This measure recognized hedging effects but ignored diversification benefits, like the CAD measure. However, many commentators sated that the building block VaR measure was not sophisticated enough. Many banks were already implementing proprietary VaR measures inside firms. Most of them took diversification benefits into consideration, and some of them even recognized portfolios' non-linearity properties.

In April 1995, the committee launched an amended proposal. This made a number of amendments, including the extension of market risk capital requirements to cover organization-wide commodities exposures. A key provision allowed banks to use either a regulatory building-block VaR measure or their own proprietary VaR measure for computing capital requirements. Nevertheless, the adoption of a proprietary measure required approval from regulators. A bank, which wants to use its own model, have to set independent risk management function and satisfy regulators that it was align with acceptable risk management practices. Proprietary VaR measure needs to support a 10-day 99% VaR metric, stricter than the building block measure, and be able to recognize the non-linear exposures of options. Diversification benefits could be taken into account within broad asset categories- equity, fixed income, foreign exchange and commodities- but not across asset categories. Market risk capital requirements were set equal to the greater of the previous day's VaR, or the three times of the average VaR over the previous six days.

The alternative building block measure- called the "standardized measure" right now- was changed from the proposal in 1993. Risk weightings remained the

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[3] International Organization of Securities Commissioners (IOSCO) was founded in 1974 to promote the development of Latin American securities markets. In 1983, its focus was expanded to encompass securities markets around the world.
[4] Capital Adequacy Directive (CAD) established uniform capital standards applicable to both universal banks' securities operations and non-bank securities firms.

same, so it may reasonably be explained as still reflecting a 10-day 95% VaR metric. Extra capital charges were added in order to recognize non-linear exposure. And the Basle Committee's new proposal was incorporated into amendment to the 1988 accord, which was adopted in 1996 and went into effect in 1998.

2.3 APPLICATION OF VAR: ECONOMIC CAPITAL

Economic Capital of Life Insurance Companies Report of Society of Actuaries (2008) stated economic capital (EC) is taking on importance within the insurance industry. The term "economic capital" is typically used to refer to a measure of required capital under an economic accounting conventions- where assets and liabilities are determined using economic principles. It would perhaps be more clearly referred to as "required economic capital." And EC is an internal calculation of the capital required, based on the company's view of risk, with calculations based on economic principles. Broadly speaking, EC is an amount of capital required calculated to give a specified level of security to policyholders in relation to the payment of their policy benefits.

And the measure of risk tolerance varies over regions. But mainly respondents of the report adopt Value-at-Risk (VaR), Tail Value-at-Risk (TVaR), and conditional tail expectation (CTE). For the North American life insurers, the use of CTE is significantly higher. The report also mentioned the correlation matrix is the most prevalent methodology for aggregating risk in North American life insurance company (47%).

In the report, SOA demonstrated the pros and cons of existing EC methodologies. In terms of risk measure, they covered VaR and CTE. For VaR, conceptually, it is relatively simple to understand and use. It is widely known and used, especially in the banking industry, and is the approach favored in Europe under Solvency II. VaR is also generally consistent with the majority of the calibration data available from rating agencies, which tends to focus more on the probability of default rather than the loss given default. For modelling approaches, report also pointed out risk aggregation, which is the final step in calculating EC. To aggregate the risk, we must consider the correlations between risks. The report stated determining the correlation assumptions can be a subjective process. Lack of available data may make techniques for determining parameters impractical. For risks related to market risk or credit risk, it is possible to find historical data that can be analyzed and used to quantify correlations. However, even then, there may be insufficient historical data to determine the tail correlations with a large degree of confidence.

2.4 LIMITATION OF BANKS' MODEL

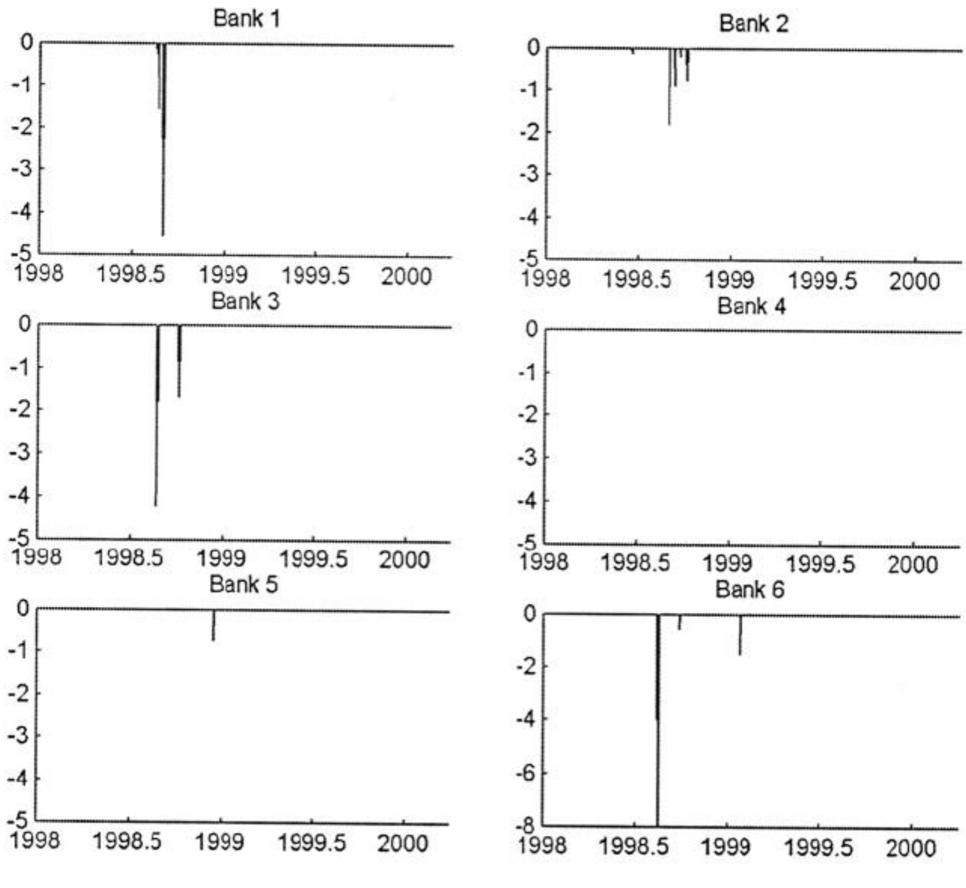


Figure 1. VaR Exceedences from Six Major Commercial Banks

Sources: Berkowitz and O'Brien (2001)

Since the authorities set the VaR measure as a standard regulatory procedure, all financial institutions have to follow the rule to align the compliance and have internal models. However, in Berkowitz and O'Brien (2001), they pointed out some problems for banks' trading risk models. They found out that, unconditionally, the VaR estimates tend to be conservative – they have fewer than expected violations-relative to the 99th percentile of P&L. However, at times losses can substantially exceed the VaR and such events tend to be clustered. Figure 1 reuses a picture from their article that shows the VaR exceedences from the six banks reported in standard deviations of the portfolio returns. It shows that the exceedences are large and appear to be clustered in time across banks. The majority of violations appear to take place during the August 1998 Russia default and ensuing Long-Term Capital Management (LTCM) debacle. This suggests that the banks' models, besides a tendency toward conservatism, have difficulty forecasting changes in the volatility of P&L.

Moreover, the empirical performance of current models reflects difficulties in structural modelling when the portfolios are large and complex. Large trading portfolios have exposures to several thousand market risk factors, with individual positions numbering in tens of thousands. It is almost impossible to output daily VaRs that measures the joint distribution of all material risks conditional on current information. To estimate the portfolio's risk structure, the banks make many approximations and parameters are often estimated only roughly. While this may appear to give representation to a wide range of potential risks, the various compromises tend to reduce any forecasting advantage.

The limitations of structural modelling extend to capturing time-varying volatility. None of the structural-based models makes any systematic attempt to capture time variation in the variances and covariances of market risks. As for evaluating exposure to liquidity or other market crises, banks are mostly limited to performing stress exercises on their portfolios.

2.5 REDUCED-FORM METHOD

Berkowitz and O'Brien (2001) claimed the clustering of violations suggests that the volatility of P&L may be time varying to a degree not captured by the bank's internal models. To adjust this and predict the volatility, they formulate an alternative VaR model determined from an ARMA(1,1) plus GARCH(1,1) model of portfolio returns. They reduced the risk factors to a univariate time series, and their reduced-form model offers a more tractable approach to estimating P&L mean and volatility dynamics. While the reduced-form approach does not account for changes in portfolio composition, they claimed that limitation can be relaxed by estimating

GARCH effects for historically simulated portfolio returns to current positions, rather than historically observed returns.

This was note the debut of reduced-form VaR forecasting approach in the academic society. Zangari(1997) suggested it in the *RiskMetrics Monitor*. And, Lopez and Walter (1999) reported a favorable results applying GARCH to portfolio returns as against applying GARCH at the risk factor level. Engle and Manganelli (1999) suggested reduced-form forecasting alternatives to GARCH.

The advantage of fitting the time series model to reported P&L is that any systematic errors in the reported numbers are incorporated into the model. This would provide the reduced-form model an advantage over the banks' internal models if the latter were not calibrated to reflect reported P&L. This following article adopts this reduced form model with the historical data of one life insurance company in Taiwan to testify its forecasting ability.

Chapter 3: Data Description

3.1 DAILY TRADING PROFIT AND LOSS

This paper collects daily profit and loss (P&L) associated with trading activities from one insurance company from Jan 1, 2011 to Feb 27, 2014 in Taiwan. The trading revenue is based on position values recorded at the close of day and, unless reported otherwise, represents the insurance company's consolidated trading activities. These activities include trading in interest rate, foreign exchange, and equity assets, liabilities, and derivatives contracts. Trading revenue includes gains and losses from daily marking to market of positions. However, for the financial products like bond, which does not mark to market daily, we use the theoretical P&L calibrated by the company.

Summary statistics for daily P&L from Jan 1, 2011 through Feb 27, 2014 are reported in Table 2. During this period, the company had negative average profits. In column 5, the Kurtosis estimate is large relative to Normal distribution, i.e. 3. Both the skewness estimated in Table 2 and the histograms in Figure 2 suggest that the portfolio returns tend to be left-skewed. The histogram of P&L in Figure 2 also exhibits extreme outliers in left tail. We take a close look at the outlier, it happened in June 30, 2011. And we both find out the P&L in 2011 are really volatile compared to 2012 and 2013. The possible reason is that the investments of the company are affected by the European debt crisis in 2009 and the following effects in this period.

Table 2. Daily P&L Summary Statistics

Obs	Mean	STD	99^{th} percentile	Kurtosis	Skew
777	-0.07	1.00	-2.34	28.22	-1.76

Notes: Daily profit and loss data are divided by its sample standard deviation to protect company's confidentiality.

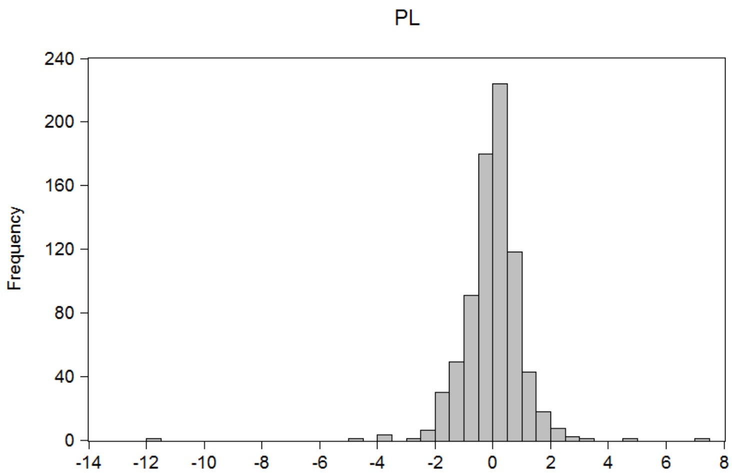


Figure 2. Daily P&L Distributions

Notes: Histogram of daily profit and loss data reported by insurance company from 1/Jan/2011 to 27/Feb/2014. Data are de-meaned and divided by its standard deviations.

3.2 DAILY VAR

The daily VaR estimates are generated by insurance company for the purpose of forecast evaluation or back-testing and are required by regulation to be calculated with the same risk model used for in internal measurement of trading risk. Generally, the VaRs are for one-day ahead horizon and a 99% confidence level for losses. However, this paper also test a 95% confidence level VaR. Since, there are only few violations of 99% confidence interval VaR. With statistical concerns, we choose 95% confidence level VaR to have more observation units. In our case, the insurance company's internal model with a VaR confidence level of 99% only has four exceptions during the sample period. With 95% confidence interval VaR, the internal model has 19 exceptions during this period.

At 95^{th} and 99^{th} percentile, P&L would be expected to exceed VaR 38 and 7 times in 777 trading days. However, the numbers of violation are only 19 and 4 times in this period. With this sense, the internal VaR forecasts happen to be conservative. We can drill down this phenomenon further by looking at the mean violation at Table 3. Column 4 shows that the mean violations of 95% and 99% VaR are more than one and two standard deviations beyond the VaR. To get a sense of the size of these violations, we take Normal distribution as a benchmark. Under a Normal distribution the probability of a loss just one standard deviation beyond a 99% VaR is 0.04%.

And the probability of a loss two standard deviations beyond 99% VaR is virtually 0. With that in mind, while violations of VaR are infrequent, the magnitudes of violations can be surprisingly large. In Figure 3, we present the time series of insurance company's P&L and corresponding one-day ahead 95^{th} and 99^{th} percentile VaR forecast (expressed in terms of the standard deviation of the insurance company's P&L). This plot tends to confirm the conservativeness of the VaR forecasts where violations of VaR are relatively few but large.

Table 3. Daily VaR Summary Statistics

Confidence Interval	Mean VaR	Number of Violation	Mean Violation
95%	-2.07	19	-1.03
99%	-3.10	4	-2.78

Notes: Daily VaR data are divided by its sample standard deviation to protest the confidentiality. Mean violation refers to the loss in excess of the VaR.

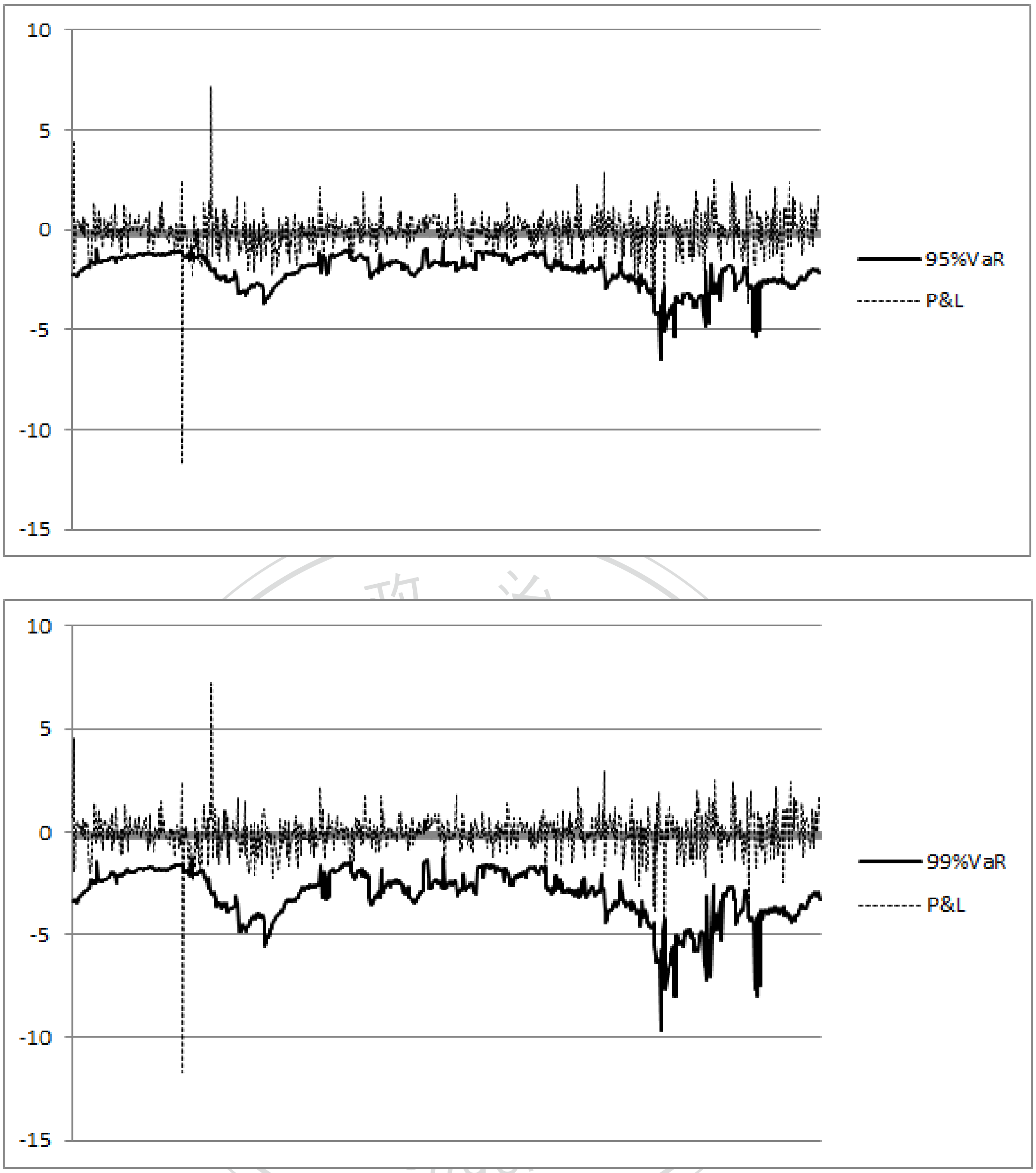


Figure 3. Internal Daily 95% & 99% VaR Models and Actual P&L

Notes: The upper model is used to forecast the one-day ahead 95% percentile of P&L. The lower model is used to forecast the one-day ahead 99% percentile of P&L. Daily P&L are plotted by dotted lines, and VaR are plotted by lines. Data are expressed in standard deviations.

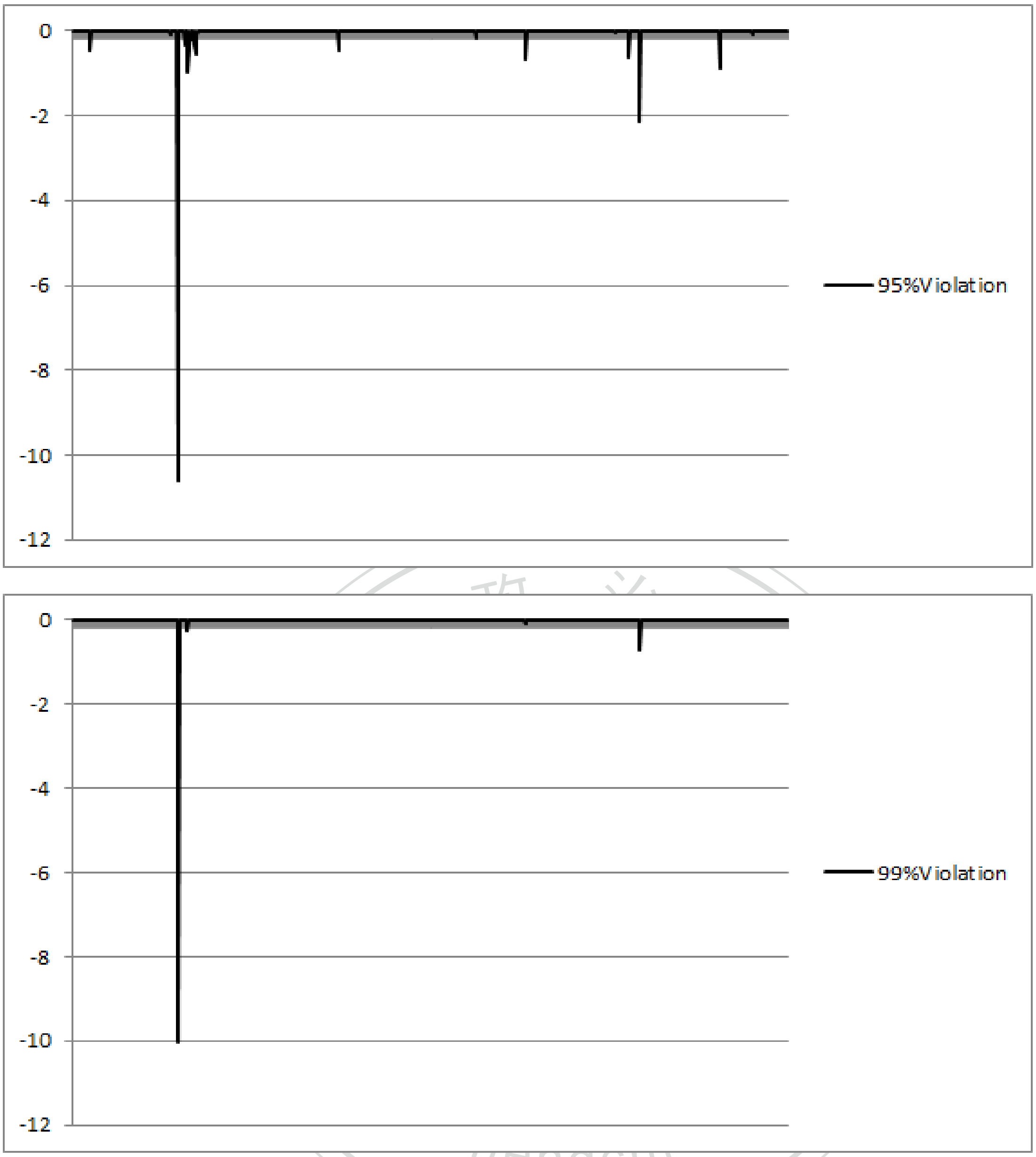


Figure 4. Violations of Internal 95% & 99% VaR Models

Note: The upper plot shows the daily P&L for those days on which P&L drops below the forecast 95th percentile given the internal models, and the lower one shows the 99th percentile. Data are expressed in standard deviations.

Chapter 4: Research Method

4.1 VALUE-AT-RISK (VAR)

Jorion (2001) mentioned two ways of computing VaR. They are nonparametric and parametric VaR. In this paper, we use the parametric method to compute VaR. We pick a normal distribution to fit the data. First, we need to translate the general distribution f(w) into standard normal distribution$\Phi(\varepsilon)$, where ε has mean zero and standard deviation of unity. We associate W* with cutoff return R* such that W*=(1+R*). Generally, R* is negative and can be written as -|R*|. Further, we can associate R* with standard normal deviate $\alpha > 0$ by setting

$$-\alpha = \frac{-|R^*|-\mu}{\sigma} \tag{1-1}$$

It is equivalent to set

$$1-c = \int_{\infty}^{W^*} f(w)\,dw = \int_{-\infty}^{-|R^*|} f(r)dr = \int_{-\infty}^{-\alpha} \Phi(\epsilon)\mathrm{d}\epsilon \tag{1-2}$$

Thus the problem of finding VaR is equivalent to finding the deviate α such that the area to the left of it is equal to 1-c. For a defined probability p, the deviate α can be found from table of cumulative standard normal distribution function, that is,

$$p = N(x) = \int_{-\infty}^{x} \Phi(\epsilon)\mathrm{d}\epsilon \tag{1-3}$$

We then retrace our steps, back from α we just found to cutoff return R* and VaR. From equation (1-1), the cutoff return is

$$R^* = -\alpha\sigma + \mu \tag{1-4}$$

From more generality, assume now that the parameters μ and σ are expressed on the annual basis. The time interval considered is Δt, in years. We find the VaR relative to the mean as

$$VaR(mean) = -W_0(R^* - \mu) = W_0\alpha\sigma\sqrt{\Delta t} \tag{1-5}$$

In other words, the VaR figure is simply a multiple of the standard deviation of the distribution times an adjustment factor that is related directly to the confidence level and horizon. When VaR is defined as an absolute dollar loss, we have

$$VaR(zero) = -W_0R^* = W_0(\alpha\sigma\sqrt{\Delta t} - \mu\Delta t) \tag{1-6}$$

Set the return r_t, the 99% VaR forecast is then given by $\hat{r}_{t+1} - 2.33\hat{\sigma}_{t+1}$, and the 95% VaR forecast is given by $\hat{r}_{t+1} - 1.96\hat{\sigma}_{t+1}$.

4.2 TIME SERIES MODEL

Figure 4 shows the violations of 95% VaR tend to be clustered[5]. That suggests the volatility of P&L may be time varying to a degree not captured by the internal models. To capture and predict the volatility, we formulate an alternative VaR model determined from time series models of portfolio return. Time series models allow us to have r_t and σ_t; hence, we can use delta-normal method to compute daily VaR for the trading positions.

[5] With total 19 violations, there are 8 violations happened in July 2011. And among them, there are 4 violations followed by previous-day violation.

4.2.1 The ARMA Process

We need sample mean to compute VaR. In this section, we will introduce ARMA process to generate the mean we need. Enders (2010) demonstrates the ARMA process with white-noise at the beginning part. A sequence $\{\varepsilon_t\}$ is a white-noise process if each value in the sequence has a mean of zero, has a constant variance, and is uncorrelated with all other realizations. Formally, if the notation E(x) denotes the theoretical mean value of x, the sequence ε_t is a white-noise process if, for each time period t,

$E(\varepsilon_t) = E(\varepsilon_{t-1}) = \cdots = 0$

$E({\varepsilon_t}^2) = E({\varepsilon_{t-1}}^2) = \cdots = \sigma^2$ [or var(ε_t)=var(ε_{t-1})=…=σ^2]

$E(\varepsilon_t \varepsilon_{t-s}) = E\left(\varepsilon_{t-j}\varepsilon_{t-j-s}\right)$

=0 for all j and s [or $cov(\varepsilon_t \varepsilon_{t-s}) = cov\left(\varepsilon_{t-j}\varepsilon_{t-j-s}\right) = 0$]

Have white-noise in mind, for each period t, x_t is constructed by taking the values $\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_{t-q}$ and multiplying each by the associated value of β_i. A sequence formed in this manner is called a mobbing average of order q and is denoted by MA(q).

$$x_t = \sum_{i=0}^{q} \beta_i \, \varepsilon_{t-i} \tag{2-1-1}$$

It is possible to combine a moving-average process with a linear difference equation to obtain an autoregressive moving-average model. Consider the pth order difference equation

$$y_t = a_0 + \sum_{i=1}^{p} a_i \, y_{t-i} + x_t \tag{2-1-2}$$

Now let $\{x_t\}$ be the MA(q) process given by (2-1-1), so that we can write

$$y_t = a_0 + \sum_{i=1}^{p} a_i\, y_{t-i} + \sum_{i=0}^{q} \beta_i\, \varepsilon_{t-i} \tag{2-1-3}$$

We follow the convention of normalizing units so that β_0 is always equal to unity. If the characteristic roots of (2-1-3) are all in the unit circle, $\{y_t\}$ is called an autoregressive moving-average (ARMA) model for y_t. If we take ARMA(1,1) as an example, we can write the equation as following and take y_t as r_t.

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t + \beta_1 \varepsilon_{t-1}$$

4.2.2 The ARCH/ GARCH Process

With mean at hand, we still need standard deviation to compute daily VaR. In this section, we will introduce two time series process which can generate time-varying volatility for the P&L. They are ARCH and GARCH processes.

ARCH

Engle (1982) let $\{\hat{\varepsilon}_t\}$ denote the estimated residuals from the model $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$ so that the conditional variance of y_{t-1} is

$$var(y_{t+1}|y_t) = E_t[(y_{t+1} - a_0 - a_1 y_t)^2] = E_t(\varepsilon_{t+1})^2$$

To this point, we have set $E_t(\varepsilon_{t+1})^2$ equal to the constant σ^2. Suppose that the conditional variance is not constant. One simple strategy is to forecast the conditional variance as an AR(q) process using squares of the estimated residuals

$$\hat{\varepsilon}_t{}^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1}{}^2 + \alpha_2 \hat{\varepsilon}_{t-2}{}^2 + \cdots + \alpha_q \hat{\varepsilon}_{t-q}{}^2 + v_t \tag{2-2-1}$$

where v_t is a white-noise process.

If the values of $\alpha_1, \alpha_2, \dots, \alpha_n$ all equal zero, the estimated variance is simply the constant α_0. Otherwise, the conditional variance of y_t evolves according to the autoregressive process given by (2-2-1). As such, you can use (2-2-1) to forecast the conditional variance at t+1 as

$$E_t \hat{\varepsilon}_t{}^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_t{}^2 + \alpha_2 \hat{\varepsilon}_{t-1}{}^2 + \cdots + \alpha_q \hat{\varepsilon}_{t+1-q}{}^2$$

For this reason, an equation like (2-2-1) is called an autoregressive conditional heteroskedastic (ARCH) model. So, ARCH(1) can be expressed by

$$y_t | \Omega_t \sim N(x_t, a, \sigma_t{}^2)$$

$$\varepsilon_t = y_t - x_t a,$$

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}{}^2$$

among them $x_t = (1, x_{1t}, x_{2t}, \dots, x_{kt})$ and $a = (a_0, a_1, a_2, \dots, a_k)'$.

GARCH

Bollerslev (1986) extended Engle's original work by developing a technique that allows the conditional variance to be an ARMA process. Let the error process be such that

$$\varepsilon_t = v_t \sqrt{h_t}$$

where $\sigma_v{}^2 = 1$, and

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{P} \beta_i \, h_{t-i} \qquad (2\text{-}2\text{-}2)$$

Since $\{v_t\}$ is a white-noise process, the conditional and unconditional means of ε_t are equal to zero. Taking the expected value of ε_t, it is easy to verify that

$$E(\varepsilon_t) = E\left[v_t(h_t)^{\frac{1}{2}}\right] = 0$$

This important point is that the conditional variance of ε_t is given by $E_{t-1}\varepsilon_t{}^2 = h_t$. Thus, the conditional variance of ε_t is the ARMA process given by the expression h_t in (2-2-2). This general ARCH(p,q) model- called GARCH(p,q)- allows for both autoregressive and moving-average components in the heteroskedastic variance. Hence, GARCH(1,1) can be expressed by

$$y_t|\Omega_t \sim N(x_t, a, \sigma_t{}^2)$$

$$\varepsilon_t = y_t - x_t a,$$

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}{}^2 + \beta_1 \sigma_{t-1}{}^2$$

among them $x_t = (1, x_{1t}, x_{2t}, \dots, x_{kt})$ and $a = (a_0, a_1, a_2, \dots, a_k)'$.

Now, we understand the ARMA and ARCH/GARCH processes, and we can combine two processes to generate the mean and standard deviation for computing VaR. Take ARMA(1,1)- GARCH(1,1) as an example, it can be represented by the following equations

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t + b_1 \varepsilon_{t-1},$$

$$\varepsilon_t \sim N(0, \sigma_t{}^2),$$

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}{}^2 + \beta_2 \sigma_{t-1}{}^2$$

The $\mathrm{y_t}$ stands for mean, r_t, and σ_t stands for standard deviation. With these two series, we can compute 95% VaR forecast at time t by $\hat{r}_{t+1} - 1.96\hat{\sigma}_{t+1}$ and 99% VaR forecast by $\hat{r}_{t+1} - 2.33\,\hat{\sigma}_{t+1}$.

4.3 MODEL SELECTION

After fitting P&L with time series models, we have to pick the best among them. And, we use AIC and SBC as the standard to verify the fitting performance of models.

The AIC and the BIC

In Enders (20100, for a given sample size T, selecting the values of p and q so as to minimize AIC (Akaike Information Criterion) equivalent to selecting p and q so as to minimize the sum:

$$AIC = T \ln(SSR)+2(1+p+q)$$

Minimizing the value of the AIC implies that each estimated parameter entails a benefit and a cost. Clearly, a benefit of adding another parameter is that the value of SSR is reduced. The cost is that degrees of freedom are reduced and there is added parameter uncertainty. Thus adding additional parameters will decrease ln(SSR) but will increase (1+p+q). The AIC allows you to add parameters until the marginal cost (i.e., the marginal cost is 2 for each parameter estimated) equals the marginal benefit.

The BIC (Schwartz Baysian Information Criterion) incorporates the larger penalty (1+p+q) lnT. To use the BIC, select the values of p and q so as to minimize

$$BIC = T \ln(SSR) + (1+p+q) \ln(T)$$

For any reasonable sample size, $\ln(T) > 2$ so that the marginal cost of adding parameters using the BIC exceeds that of the AIC. Hence, the BIC will select a more parsimonious model than the AIC. As indicated in the text, the BIC has superior large simple properties. It is possible to prove that the BIC is asymptotically consistent while the AIC is biased toward selecting an overparameterized model. However, Monte Carlo studies have shown that in small samples, the AIC can work better than the BIC.

4.4 BACK TESTING

In order to compare the performance of models, we have to do the backtesting to verify the accuracy of VaR models. Backtesting is a formal statistical framework that consists of verifying that actual losses are in line with projected losses. This involves systematically comparing the history of VaR forecasts with their associated portfolio return.

4.4.1 Kupiec

Kupiec (1995) develops approximate 95 percent confidence regions for verification test, which are reported in Table 4. These regions are defined by tail points of the log-likelihood ratio:

Table 4. Model Backtesting, 95% Non-rejection Test Confidence Regions

Probability level p	VaR Confidence Level c	Nonrejection Region for Number of Failures N		
		T= 252 Days	T= 510 Days	T= 1000Days
0.01	99%	N < 7	1 < N < 11	4 < N < 7
0.025	97.5%	2 < N < 12	6 < N < 21	15 < N < 36
0.05	95%	6 < N < 20	16 < N < 36	37 < N < 65
0.075	92.5%	11 < N < 28	27 < N < 51	59 < N < 92
0.10	90%	16 < N < 36	38 < N < 65	81 < N < 120

Note: N is the number of failures that could be observed in a sample size T without rejecting the null hypothesis that p is the correct probability at the 95 percent level of test confidence.
Source: Adapted from Kupiec (1995)

$$LR_{uc} = -2\ln[(1-p)^{T-N}p^{N}] + 2\ln\{[1-(N/T)]^{T-N}(N/T)^{N}\}$$

which is asymptotically, (i.e., wheh T is large) distributed Chi-square with one degree of freedom under the null hypothesis that p is the true probability. Thus we would reject the null hypothesis if LR > 3.841.

4.4.2 Christoffersen

Christoffersen (1998) extends the LR_{uc} statistic to specify that the deviation must be serially independent. The test is set up as following:

Deviation indicator= 0 if VaR is not exceeded;

Deviation indicator= 1 otherwise.

Then we define T_{ij} as the number of days in which state j occurred in one day while it was at I the previous day andπ_i as the probability of observing an exception conditional on state i the previous day. Table 5 shows how to construct a table of conditional exceptions.

If today's occurrence of an exception is independent of what happened the previous day, the entries in the second and third columns should be identical. The relevant test statistic is

$$LR_{ind} = -2\ln\left[(1-\pi)^{(T_{00}+T_{10})}\pi^{(T_{01}+T_{11})}\right] + 2\ln[(1-\pi_0)^{T_{00}}{\pi_0}^{T_{01}}{\pi_1}^{T_{11}}]$$

Here, the first term represents the maximized likelihood under the hypothesis that exceptions are independent across days, or $\pi = \pi_0 = \pi_1 = (T_{01} + T_{11})/T$.

The second term is the maximized likelihood for the observed data.

Table 5. Building an Exception Table: Expected Number of Exceptions

	Conditional		
	Day Before		
	No Exception	Exception	
Current day			
No exception	$T_{00} = T_0(1-\pi_0)$	$T_{10} = T_1(1-\pi_1)$	$T(1-\pi)$
Exception	$T_{01} = T_0(\pi_0)$	$T_{11} = T_1(\pi_1)$	$T(\pi)$
Total	T_0	T_1	$T = T_0 + T_1$

The combined test statistic for conditional coverage then is

$$LR_{cc} = LR_{uc} + LR_{ind}$$

Each component is independently distributed as $x^2(1)$ asymptotically. The sum is distributed as $x^2(2)$. Thus we should reject at the 95 precents test confidence level if LR>5.991. We would reject independence alone if LR_{ind}>3.84.

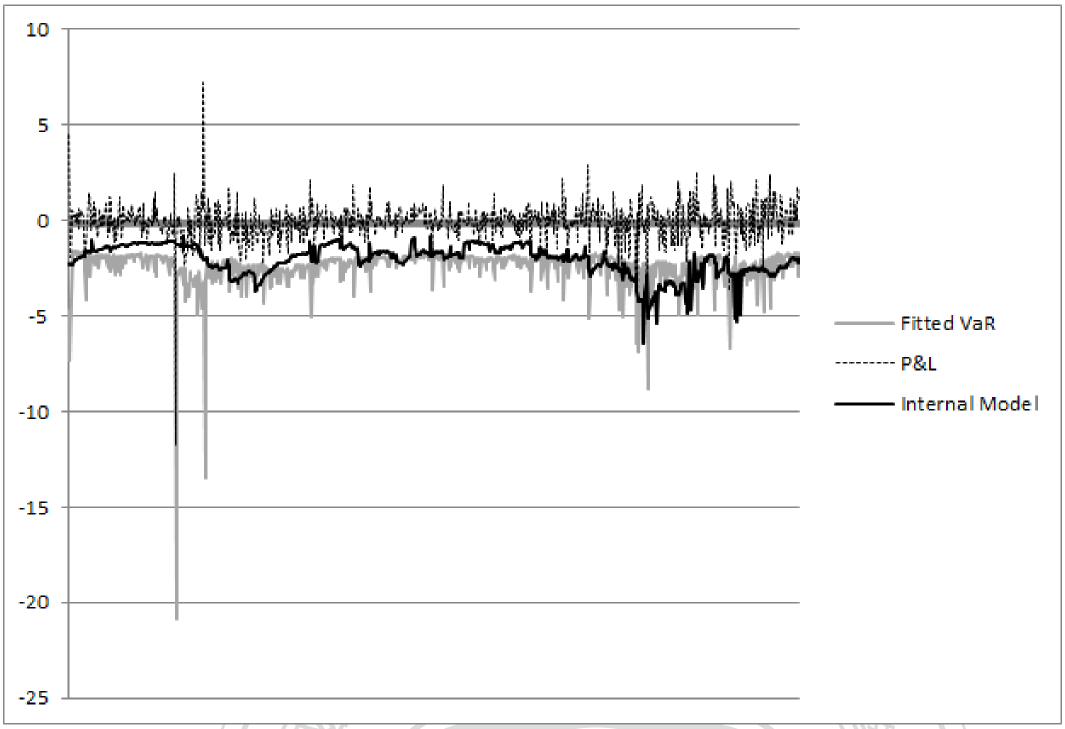
Chapter 5: Results

The time series models are estimated each day with data available up to that point. To obtain stable estimates for the model, forecasts for 2011 (days 1 through 243) are in-sample. Rolling out-of-sample forecasts starts after 2012. Out-of-sample estimates are updated daily.

Here, we adopt two kinds of reduced-form models; they are fitted model and Berkowitz & O'Brien model (BO Model). The fitted model uses the first 243-day data as the in-sample to fit a time series model; and the BO model follows Berkowitz & O'Brien (2002) using a ARMA(1,1)-GARCH(1,1) as the time series model.

Given parameters estimates, we forecast the next day's 95% and 99% VaR. The results of the forecast, both within and out-of-sample, are shown in Figure 5 by the grey line, along with P&L by the dotted line and internal model by solid line. As we can see one-day ahead reduced-form forecasts appear to track the lower tails of P&L really well compared to the internal structural model. It tracked the huge P&L drop in 2011, which did not caught by the internal model. This shows that time series model does better at adjusting in volatility through time.

95% VaR of Fitted Model



95% VaR of BO Model

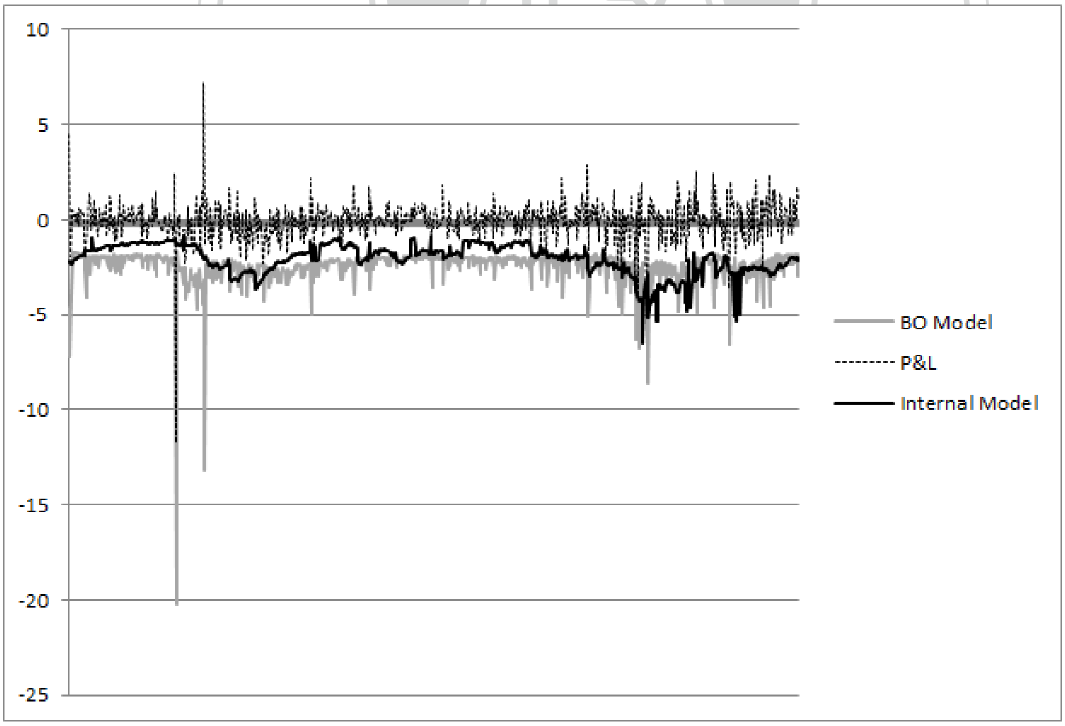
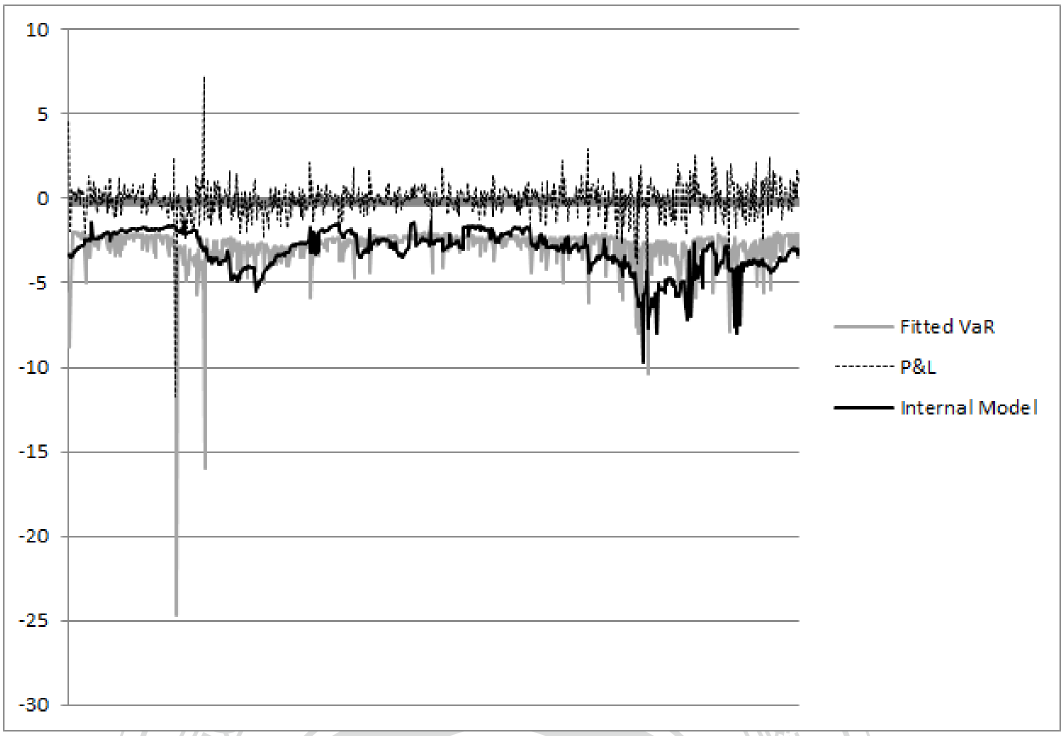


Figure 5. P&L, 95% Internal VaR, Fitted VaR, and BO VaR

Note: Data are expressed in standard deviations.

99% VaR of Fitted Model



99% VaR of BO Model

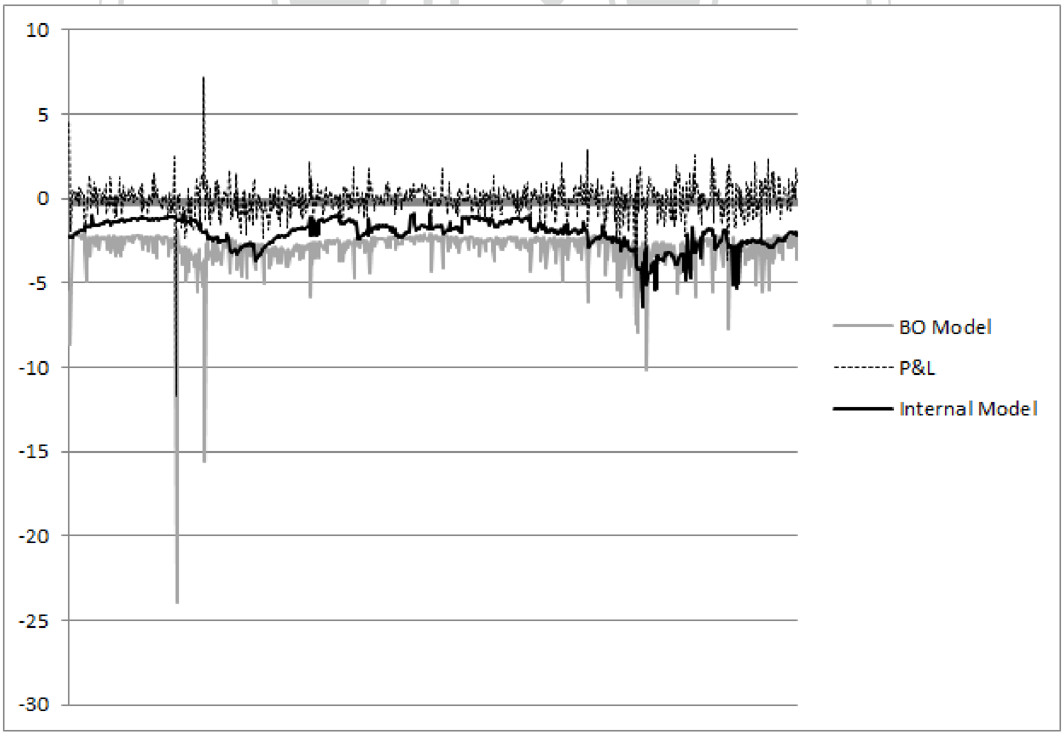


Figure 6. P&L, 99% Internal VaR, Fitted VaR, and BO VaR

Note: Data are expressed in standard deviations.

Summary statistics and backtests for three models are presented in Table 6 and Table 7 as following.

The third column of Table 6 shows that the time series models remove first-order persistence successfully. Table 6 also shows that both time series models can have lower mean VaR and max VaR in 99th percentile VaR. The phenomenon might be interpreted that time series models have better performance in the fat-tailed situation. But, in 95th percentile VaR, that phenomenon disappears.

The higher mean VaRs mean the internal models are more conservative and should generate lower mean violation and max violation. However, the mean violation and max violation, shown in column 8 and 9, exhibit that is not the case. Even though the internal model is more conservative, the time series models still can have lower mean and max violation.

This result indicates a potentially important advantage for the reduced-form model. Since the magnitudes of the VaR forecasts are used for determined economic capital for the insurance companies. The reduced-form time series models are able to deliver lower required capital requirement without having large violations. This reflects the reduced-form models have greater responsiveness to the P&L volatility.

Table 6. Summary Statistics of Three Models

Summary Statistics	Obs	Box-Ljung Stat		Mean VaR	Number Violations	Continued Violations	Mean Violation	Max Vio
Internal Model	777	na	95%	-2.07	19	4	-1.03	-10.59
			99%	-3.10	4	0	-2.78	-10.03
Fitted Model		3.00 [0.08]	95%	-2.45	9	0	-1.41	-6.93
			99%	-2.90	5	0	-1.94	-6.02
B.O. Model		2.58 [0.11]	95%	-2.48	8	0	-1.58	-7.05
			99%	-2.94	5	0	-1.96	-6.16

Note: Box-Ljung statistics are for first-order serial correlation. The Internal Model are calibrated by insurance company. In column 2, the fitted model is ARMA(1,1)-ARCH(1); in column 3, the model is ARMA(1,1)-GARCH(1,1). The grey shading parts have better performance compared to Internal Models.

Formal Backtests for three models are presented in Table 7. The backetest results provide little basis to distinguish between the time series models and internal model.

For 95th percentile VaR, all methods are rejected in terms of coverage. And, for the independence, only the 95th percentile VaR of internal model is rejected. The main reason of that is because there are 4 continuous violations coming after previos –day violation in this period. And, for 99th percentile VaRs of all methods are not rejected.

Table 7. Backtests for Three Models

Backtests		Violation Rate	Coverage	Conditional Coverage	Independence
Internal	95%	0.0245	13.05**	37.71**	24.66**
			[0.00]	[0.00]	[0.00]
	99%	0.0051	2.25	2.29	0.04
			[0.13]	[0.32]	[0.84]
Fitted Model	95%	0.0116	34.57**	34.78**	0.21
			[0.00]	[0.00]	[0.65]
	99%	0.0064	1.14	1.20	0.06
			[0.29]	[0.55]	[0.81]
B.O. Model	95%	0.0103	37.68**	37.86**	0.17
			[0.00]	[0.00]	[0.68]
	99%	0.0064	1.14	1.20	0.06
			[0.29]	[0.55]	[0.81]

Note: P-values are demonstrated in square brackets. The 5% critical value is 3.84, and the 1% critical value is 6.64. * and ** stand for significance at the 5 and 1 percent levels, respectively.

To have a further understanding of this reduced-form model, this paper separates the data into three parts, 2011, 2012, and 2013. And the results of these models are listed in Table 8 to Table 13 as following.

We follow the same fashion with the previous method. Using fitted model and B.O. model as reduced-form time series models. For both models, we use the first half data as in-sample and the rest as out-of-sample. Take 2011's data as an example, we have 243 P&L data, so we use the first 120 data to fit time series models for fitted model and B.O. model.

As we can see, in Table 8, the fitted models barely have violations, and the mean VaRs are bigger than internal models in both 95th and 99th percentile VaRs. Even though they have lower max violations, they have greater mean violations too. That means the fitted models do not outperform the internal models. In addition, the backtests in Table 9 also shows that the 95th percentile VaR of fitted model is the only model rejected in the coverage test in 2011.

And for the B.O. model in 2011, it has the same result as in all samples. 99th percentile VaR has a better performance compared to the internal model, and it passes both coverage and independence tests in backtests.

Table 8. Summary Statistics of Three Models in 2011

Summary Statistics	Obs	Box-Ljung		Mean VaR	Number Violation	Continued Violations	Mean Violation	Max Vio
Internal Model	243	na	95%	-1.53	11	4	-1.06	-8.57
			99%	-2.29	2	0	-4.17	-8.12
Fitted Model		1.97 [0.16]	95%	-2.41	2	0	-2.50	-5.02
			99%	-2.84	1	0	-4.20	-4.20
B.O. Model		8.22 [0.00]	95%	-1.72	7	0	-1.09	-6.40
			99%	-2.05	3	0	-2.02	-5.92

Note: Box-Ljung statistics are for first-order serial correlation. The Internal Models are calibrated by insurance company. In column 2, the Fitted Model is ARCH(1); in column 3, B.O. Model is ARMA(1,1)-GARCH(1,1). The grey shading parts have better performance compared to Internal Model.

Table 9. Backtests of Three Models in 2011

Backtests		Violation Rate	Coverage	Conditional Coverage	Independence
PL11	95%	0.045	0.12	12.49**	12.37**
			[0.73]	[0.00]	[0.00]
	99%	0.008	0.08	0.11	0.03
			[0.78]	[0.95]	[0.86]
Fitted Model	95%	0.008	13.52**	13.55**	0.03
			[0.00]	[0.00]	[0.86]
	99%	0.004	1.09	1.10	0.03
			[0.30]	[0.58]	[0.86]
B.O. Model	95%	0.029	2.69	3.11	0.42
			[0.10]	[0.21]	[0.52]
	99%	0.012	0.13	0.20	0.08
			[0.72]	[0.90]	[0.78]

Note: P-values are demonstrated in square brackets. The 5% critical value is 3.84, and the 1% critical value is 6.64. * and ** stand for significance at the 5 and 1 percent levels, respectively.

And in 2012, the results in summary statistics have a different fashion. In Table 10, the fitted models still barely have violations. This time, compared to internal models, fitted model and B.O. model have better performance in the 95th percentile VaRs in terms of mean VaR, mean violation, and max violation. This fashion is totally different form the total sample and sample in 2011- 99th percentile VaRs of most models have better performances. Moreover, in Table 11, B.O. model passes the coverage test in 95th percentile VaR in backtest.

Table 10. Summary Statistics of Three Models in 2012

Summary Statistics	Obs	Box-Ljung		Mean VaR	Number Violation	Continued Violations	Mean Violatio	Max Vio
Internal Model	249	na	95%	-2.64	3	0	-0.73	-1.12
			99%	-3.95	1	0	-0.18	-0.18
Fitted Model		2.44 [0.12]	95%	-2.19	2	0	-0.48	-0.88
			99%	-2.57	1	0	-0.50	-0.50
B.O. Model		1.04 [0.31]	95%	-2.15	7	0	-0.29	-0.75
			99%	-2.48	2	0	-0.40	-0.52

Note: Box-Ljung statistics are for first-order serial correlation. The Internal Models are calibrated by insurance company. In column 2, fitted model is ARMA(3,3); in column 3, B.O. Model is ARMA(1,1)-GARCH(1,1). The grey shading parts have better performance compared to Internal Models.

Table 11. Backtests of Three Models in 2012

Backtests		Violation Rate	Coverage	Conditional Coverage	Independence
PL12	95%	0.012	10.73** [0.00]	10.80** [0.00]	0.07 [0.79]
	99%	0.004	1.16 [0.28]	1.17 [0.56]	0.01 [0.92]
Fitted Model	95%	0.008	14.04** [0.00]	14.07** [0.00]	0.03 [0.86]
	99%	0.004	1.16 [0.28]	1.17 [0.56]	0.01 [0.93]
B.O. Model	95%	0.028	2.96 [0.09]	3.00 [0.22]	0.04 [0.84]
	99%	0.008	0.10 [0.75]	0.14 [0.93]	0.03 [0.86]

Note: P-values are demonstrated in square brackets. The 5% critical value is 3.84, and the 1% critical value is 6.64. * and ** stand for significance at the 5 and 1 percent levels, respectively.

In the last year, expressed in Table 12, the summary statistics of all reduced-form models do not outperform the internal models in either percentile.

And, we can tell that fitted model and BO model have relatively more violations in 99th percentile VaRs compared to the previous tests. As the result, both of them cannot pass the coverage tests.

In Table 13, we can see the violation rates of fitted model and the BO model in 99th percentile VaRs are obliviously over 1%.

Table 12. Summary Statistics of Three Models in 2013

Summary Statistics	Obs	Box-Ljung		Mean VaR	Number Violation	Continued	Mean Violatio	Max Vio
Internal Model	248	na	95%	-2.53	5	0	-0.71	-2.00
			99%	-3.78	1	0	-0.67	-0.67
Fitted Model		1.46 [0.23]	95%	-1.71	11	0	-1.00	-3.06
			99%	-2.01	8	0	-1.03	-2.77
B.O. Model		0.20 [0.65]	95%	-1.84	9	0	-1.02	-2.95
			99%	-2.19	7	0	-0.91	-2.62

Note: Box-Ljung statistics are for first-order serial correlation. The Internal Models are calibrated by insurance company. In column 2, the Fitted Model is ARCH(1); in column 3, the B.O. Model is ARMA(1,1)-GARCH(1,1).

Table 13. Backtests of Three Models in 2013

Backtests		Violation Rate	Coverage	Conditional Coverage	Independence
PL13	95%	0.020	5.95** [0.01]	6.16* [0.05]	0.21 [0.65]
	99%	0.004	1.15 [0.28]	1.16 [0.56]	0.01 [0.92]
Fitted Model	95%	0.044	0.17 [0.68]	1.19 [0.55]	1.02 [0.31]
	99%	0.032	7.82** [0.01]	8.36** [0.02]	0.53 [0.47]
B.O. Model	95%	0.036	1.08 [0.30]	1.76 [0.42]	0.68 [0.41]
	99%	0.028	5.57* [0.02]	5.98* [0.05]	0.41 [0.52]

Note: P-values are demonstrated in square brackets. The 5% critical value is 3.84, and the 1% critical value is 6.64. * and ** stand for significance at the 5 and 1 percent levels, respectively.

In the last, we try different ways to compute VaR for time series method. First, we try moving window method (M.W. method). In this method, we use the first 243 samples as the in-sample to fit the time series model, and we forecast the first half year of 2012. Following that, we move the data window to 120-367 as in-sample to fit the second time series model, and we forecast the second half year of 2012.

Second, we use the data in 2011 as the in-sample to fit a time series model and forecast the VaR in 2012 (2011 Forecast). Last, we use the 2011 data as the in-sample to fit the Berkowitz and O'Brien's ARMA(1,1)-GARCH(1,1) model and forecast the VaR in 2012 (2011 B.O. Forecast). The results are demonstrated in the Table 14 and 15.

In Table 14, we can see that the MW method can use a lower mean VaR to generate lower mean violation compared to internal model in 99^{th} percentile VaR. And the mean violation is the lowest in all method we have tried in this paper. In addition, it passes both backtests. However, it cannot generate a lower max violation at the same time. For 2011 forecast and 2011 B.O. forecast models, both of them do not have violation in 99^{th} percentile VaR and do not outperform in 95^{th} percentile VaR.

Table 14. Summary Statistics for Internal Model and Other Methods

Summary Statistics	Obs		Mean VaR	Number Violations	Continued Violations	Mean Violation	Max Vio
Internal Model	249	95%	-2.64	3	0	-0.73	-1.12
		99%	-3.95	1	0	-0.18	-0.18
M.W. Method		95%	-3.27	2	0	-0.58	-0.66
		99%	-3.87	2	0	-0.10	-0.20
2011 Forecast		95%	-3.66	1	0	-0.05	-0.05
		99%	-4.33	0	0	na	Na
B.O. 2011 Forecast		95%	-3.66	1	0	-0.05	-0.05
		99%	-4.34	0	0	na	Na

Note: The Internal Models are calibrated by insurance company. The first and the second models are both ARMA(1,1)-ARCH(1) in M.W. method. The model of 2011 forecast is ARMA(1,1)-ARCH(1). The B.O. 2011 forecast model is ARMA(1,1)-GARCH(1,1).

Table 15. Backtests of Internal Model and Moving Window Method in 2012

Backtests		Violation Rate	Coverage	Conditional Coverage	Independence
PL12	95%	0.012	10.73**	10.80**	0.07
			[0.00]	[0.00]	[0.79]
	99%	0.004	1.16	1.17	0.01
			[0.28]	[0.56]	[0.92]
M.W. Method	95%	0.008	14.04**	14.07**	0.03
			[0.00]	[0.00]	[0.86]
	99%	0.008	0.10	0.14	0.03
			[0.75]	[0.93]	[0.86]
2011 Forecast	95%	0.004	1.16	1.17	0.01
			[0.28]	[0.56]	[0.93]
	99%	0.000	na	na	na
B.O. 2011 Forecast	95%	0.004	1.16	1.17	0.01
			[0.28]	[0.56]	[0.93]
	99%	0.000	na	na	na

Note: P-values are demonstrated in square brackets. The 5% critical value is 3.84, and the 1% critical value is 6.64. * and ** stand for significance at the 5 and 1 percent levels, respectively.

Chapter 6: Conclusions

This paper implements an empirical test about the methodology of VaR on a life insurance company in Taiwan. VaR is a really important equipment to quantify the risk for financial institutions nowadays. For insurance companies, they can use VaR method to generate the economic capital they need as capital buffer to survive the crisis.

The structural model, adopted by most financial institutions, might have some limitations when computing VaR. Berkowitz and O'Brien (2002) found the models used by banks tend to be conservative. However, the losses can substantially exceed the VaR and such events tend to be clustered.

Moreover, the total amount of capital invested by insurance companies has grown substantially in the past few years. The market risk factors used by internal structural models, therefore, become larger and more complex. It is almost impossible for institutions to compute daily VaR considering joint distribution conditional on the current information.

This study is the first article that uses the univariate method with historical data of one life insurance company in Taiwan and provides its performance compared to the internal models that is actually in use. This paper follows the method of Berkowitz and O'Brien (2002) adopting a reduced-form model- time series model. It considers life insurance company's trading P&L as one investment portfolio and reduces its risk factors to a univariate time series. The aim is trying to solve the aggregation problem of the internal structural model.

The followings are the general conclusions we have found in this paper,

1. On average, the time series models achieve the target violation rate in 99th percentile VaR coverage.

   At the same time, the mean violations for the time series models are lower than the internal models. This result demonstrates the reduced-form time series models generally have better performances compared to the internal models. That can be interpreted as time series models track lower bound of P&L better than the internal models, since they do better at adjusting in volatility through time. However, we do not have consistent results when we separate the data in to annual data.

2. Almost all 95th percentile VaR in every method cannot pass the coverage test in backtesting.

    This result shows that the models we are using right now are too conservative. We can view this phenomenon in two perspectives. First, in the institutional perspective, the financial institutions cannot use their capital efficiently since the required capital they have to withdraw is higher. Second, in the supervisory perspective, the supervisors will not take this issue too seriously, since the financial institutions view the required capital in a conservative way. That means financial institutions will have more capital buffer to endure the financial crises.

3. The ARMA(1,1)-GARCH(1,1) model suggested by Berkowitz and O'Brien has the best performance in all the samples.

    The reason is probably because this model considers both ARMA and GARCH model all the time, and this model fits the P&L of life insurance company really well. On the contrary, the fitted models sometimes will only have ARMA or only ARCH process in the model and have inferior forecasting performances. Hence, instead of fitting the time series model, the insurance companies can directly try ARMA(1,1)-GARCH(1,1) model to fit their P&L data.

4. The financial institutions can use the reduced-form model- time series model as a complementary method to the internal model while computing VaR for the following reasons.

    First, time series models have a better performance tracking the lower bound of P&L. Compared to the reduced-form models, the internal VaRs did not adequately reflect changes in the P&L volatility. These results may reflect substantial computational difficulties in constructing large-scale structural models of market risks for large, complex portfolios. Even the structural models permit firms to examine the effects of individual positions on market risk. Time series models may have advantage in forecasting and as equipment for identifying the shortcoming of the structural models.

Second, the time series models are really easy to compute. This timesaving specialty can do a really good job as a complement to the structural models.

Chapter 7: Suggestions

This paper suggests that if anyone wants to have a further research on this topic, he can amend the following points,

7.1 MORE OBSERVATIONS

In this paper, we only have one life insurance company's around three years data. The future research can collect more observations in terms of depth and width. In depth, the researcher can use more observations to come up with more robust results. In width, the researcher can collect data from more than one company or different types of companies, e.g. property insurance company.

7.2 CRISIS TEST

In this paper, our sample period, from Jan 1, 2011 to Feb 27, 2014, does not cover the 2008 financial crisis. If the future researcher included the crisis data, we can see the performance of the reduced-form model in the severe situation. Since VaR model aims to forecast the worst situation encountered by company, the crisis data will be a huge plus for the research.

7.3 VAR'S DRAWBACK

Since VaR is not a coherent risk measure (Artzner, 1999) and can lead to inconsistent results when aggregating capital. In order to understand the economic capital financial institutions need, we can test other risk measures, like Conditional Tail Expectation, a.k.a., CTE.

7.4 DIFFERENT FORECASTING METHOD

We have tried a lot of models and different two percentile VaRs to verify the performance of the reduced-form model compared to the internal models. Nevertheless, there are still other forecasting methods have not been tested, like aggregated method, which aggregate all the past data as in-sample to fit the time series model.

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