Analyzing Convertible Bonds: Valuation, Optimal Strategies and Asset Substitution

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Abstract

This article provides an analytic pricing formula for a callable convertible bond with consideration of tax benefits, bankruptcy costs, bond maturities, and the capital structure of the bond issuer. Our structural model allows optimal strategies for call, voluntary conversion, and bankruptcy to be endogenously determined. The numerical results predict when the call redemption, the forced conversion, the voluntary conversion, and the bankruptcy of a callable convertible bond may occur. The literature findings of late calls associated with dividend payments and tax benefits are confirmed, and the hypothesis that using convertible bonds can reduce the asset substitution problem is also validated.

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Introduction

Convertible bonds, spanning the dimensions from common stocks on the one hand to straight bonds on the other, are one of the most popular hybrid financing instruments. Most convertible bonds have call provisions, making the valuation and the determination of the optimal strategies for call and conversion more complicated. Similar to ordinary bondholders, investors of convertible bonds are entitled to receive coupon payments and principal payments, and thus the default risk of the bond issuer is also essential to the valuation of convertible bonds.

There are two approaches to pricing financial instruments subject to the default risk of the issuer in the literatures: the reduced-form approach and the structural approach. It is difficult to distinguish the hazard rate and the loss on default in the reduced-form model. On the other hand, the structural approach, which has a clear link between economic fundamentals and firm's defaults, helps to understand losses on default. Owing to poor fitting results on prices of convertible bonds by reduced-form models, Rogers (1999) has ever noted that it is hard to deal realistically with convertible bonds in the reduced-form model: "The existence of convertible bonds really forces one to consider firm value – so maybe we should go for a structural approach anyway?" Although a structural model is heavy going, it definitely brings us some useful insights of corporate financing.

The pioneered work of Merton (1974) provides a structural model and explains how the risky debt can be viewed as a European contingent claim on the value of firm's assets. He further derives the closed-form valuation by using the Black-Scholes option pricing formula. Subsequently, Black and Cox (1976) first utilize the first-passage-time approach to extend Merton's model and consider the possibility that the bond issuer may default prior to the maturity. Leland (1994) further takes the tax benefits and the bankruptcy costs into account, which are viewed as the perpetual contingent claims on the unlevered asset value of a firm. By the pricing method of a perpetual American option, he provides the closed-form pricing formulas of these contingent claims, and furthermore, he uses the smooth-pasting condition to endogenize the bankruptcy strategy. Leland and Toft (1996), based on Leland (1994), use a (single) barrier option approach and construct a stationary debt structure[1] to price a finite-maturity coupon debt with consideration of endogenous bankruptcy.

As for the valuation of convertible bonds by a structural model, Ingersoll (1977a) first uses the Black-Scholes methodology and derives the closed-form pricing formula with some simplifying assumptions. In addition, he obtains the optimal call trigger which is equal to the call price multiplied by a conversion ratio, and shows that the conversion will occur only either at the time of call or at the maturity of the bond in a perfect market. Meanwhile, Brenann and Schwartz (1977) price a more general convertible bond by the finite difference method where they solve a partial differential equation with more realistic boundary conditions. Subsequently, Brenann and Schwartz (1980) allow for stochastic interest rates and take consideration of the senior debt in the issuer's capital structure. Their numerical results suggest, in a striking manner, that for a reasonable range of interest rates, the errors from the certain interest rate model are likely to be small. For practical purposes, therefore, it may be preferable to use a simple model with the constant interest rate for valuing convertible bonds. Nyborg (1996) provides an excellent survey on the valuation of convertible bonds and reviews the reasons why firms issue convertible bonds. All of the works above focus on the case with a positive net-worth covenant in which bankruptcy is triggered when the firm's asset value falls to the total outstanding debt's principal value. Recently, Sarkar (2001) and Sarkar and Hong (2004), based on the endogenous bankruptcy framework of Leland (1994), price a callable corporate bond and analyze the call probability as well as the effective duration with consideration of tax benefits and bankruptcy costs. In addition, Sarkar (2003) explores early and late calls of convertible bonds still under the perpetual maturity setting of Leland (1994), which seems unreasonable. Moreover, Sarkar (2003) only considers the possibility of forced conversion when the call is triggered but neglects the possibility of voluntary conversion by bondholders.

This paper provides a simple but complete structural model to price a callable convertible bond with finite maturity using the pricing technique of double-barrier options, where the optimal strategies for call, voluntary conversion, and bankruptcy are endogenously determined by shareholders and bondholders. Our model not only takes tax benefits, bankruptcy costs, and bond maturities into account, but also considers the possibilities that the call, the voluntary conversion, and the bankruptcy may occur prior to the maturity of the bond. In addition, our numerical results predict that when the call redemption, the forced conversion, the voluntary conversion, and the bankruptcy of a callable convertible bond may happen. The empirical literature findings of late calls associated with dividend payments and tax benefits are confirmed in our numerical analyses, and furthermore, the hypothesis that shorter-term debts and convertible debts can be used to reduce the asset substitution agency problem is also numerically validated by our model.

The remainder of this paper is organized as follows. In Section 1, we set up the modeling framework. Section 2 is devoted to the derivation of the analytic valuation of a callable convertible bond. Next, we present some numerical analyses of the optimal strategies for call, voluntary conversion and bankruptcy, and of callable convertible bond prices in Section 3. In Section 4, the asset substitution problem associated with convertible bonds is also examined. Finally, Section 5 summarizes the article and makes concluding remarks.

1. Valuation Framework

Consider a bond issuer (or an objective firm) where the callable convertible bond is the only senior issue, which continuously pays a constant coupon flow, C, with the finite time to maturity, T, and the par value, P. The other claim of the firm is the common share. Let $V(t)$ designate the unlevered asset value of the bond issuer at time t. The dynamics of $V(t)$ on the risk-neutral filtered probability space are given by

$$dV(t) = V(t)\Big((r-q)dt + \sigma dW^{Q}(t)\Big), \tag{1}$$

where r denotes the constant risk-free interest rate,[2] q is the constant payout ratio of the issuer, σ is the constant return volatility, and W^{Q} is a risk-neutral Wiener process. Owing to Harrison and Kreps (1979) and Harrison and Pliska (1981), we start with risk-neutrality and bypass the original rate of return of the unlevered asset. Here we implicitly assume that the unlevered asset is a continuously tradable asset in a frictionless market, and there is a risk-free asset earning the risk-free interest rate without any restriction.

The unlevered asset value of the bond issuer is assumed to be independent of the capital structure of the firm. This implies the validity of Modigliani-Miller Theorem, which is a standard assumption in the structural models, such as Leland and Toft (1996). Moreover, we assume there is no incentive problem between the management and the shareholders of the objective firm. In other words, our model implies that the intention of the management is always consistent with that of the shareholders, which is to maximize the common shareholders' wealth subject to the constraints placed upon the firm.

Next, we are going to characterize the callable convertible bond of the objective firm. As the title indicates, there are only call and conversion provisions involved in the callable convertible bond where no other exotic provisions prevail. Most convertible bonds are also puttable bonds which give bondholders rights to sell their bonds back to the issuer. This flexibility, however, plays a minor role in the valuation, and we shall ignore it for simplicity. As usual, if bondholders convert convertible bonds into common shares, then they will receive a fraction γ of the unlevered asset value of the issuer. One advantage of the structural model for valuing convertible bonds is that γ captures the dilution effect of the conversion, which represents the ratio between the total converted shares and the total outstanding shares after conversion. For details, we refer Schönbucher (2003, P.266). Here we implicitly assume the conversion in our model is "block conversion", that is, all the bondholders will convert the convertible bonds into the common shares at the same time. Constantinides (1984) shows that there is at least one Nash-equilibrium in various conversion strategies and in addition, the highest value of the convertible bond in these Nash-equilibria coincides with the bond value in the block conversion case.

If the issuer of the callable convertible bonds calls back all outstanding callable convertible bonds at the same time, then all the bondholders have to immediately choose either to convert callable convertible bonds into common shares, or to receive the pre-specified call price (the redemption value), $(1+\beta)P$, where βP is the call premium of the callable convertible bonds. Here we ignore the transaction costs associated with the processes of the call and the conversion, and we also neglect the call notice period and the conversion protected period in our framework.

At the initial time, assumed to be time zero for simplicity, we suppose that the upper constant call barrier, V_{Call}^0, and the upper constant conversion barrier, V_{Con}^0, are both greater than the initial unlevered asset value of the bond issuer, $V(0)$. As soon as the unlevered asset value of the bond issuer goes up and touches either V_{Call}^0 or V_{Con}^0, then either the call of the bond issuer or the voluntary conversion of the bondholders is triggered. Therefore, two first passage times can be further defined as $\tau_{Call}^0 \equiv \inf\left(t > 0 : V(t) \geq V_{Con}^0\right)$ and $\tau_{Con}^0 \equiv \inf\left(t > 0 : V(t) \geq V_{Con}^0\right)$, where τ_{Call}^0 and τ_{Con}^0 are the time that the bond issuer decides to call back the bonds, and the time that the bondholders determine to voluntarily convert the bonds into the common shares, respectively.

In addition to the results of being called or being voluntarily converted, there are still two other possible outcomes for callable convertible bonds. One is that the bond issuer declares bankruptcy prior to the time of the call, the time of the voluntary conversion, and the maturity of the bond; the other one is that callable convertible bonds mature and none of the call, the voluntary conversion and the bankruptcy occurs. Subsequently, another lower constant bankruptcy barrier is defined as V_B^0, which is less than $V(0)$. As soon as the unlevered asset value of the bond issuer goes down and touches V_B^0, the bankruptcy of the bond issuer is triggered. Once the bond issuer declares bankruptcy, the bondholders receive the recovery value, $(1-\alpha)V_B^0$, at the time of default, where α, between 0 and 1, is the ratio of bankruptcy costs or restructuring costs. Again, another first passage time can be denoted as $\tau_B^0 \equiv \inf\left(t > 0 : V(t) \leq V_B^0\right)$, where τ_B^0 is the time that the bond issuer announces bankruptcy. In the next section, we will endogenously determine the optimal strategies for call, voluntary conversion, and bankruptcy by taking the desired objectives of the bond issuer and the bondholders into consideration.

2. Pricing Callable Convertible Bonds with Default Risk

Since the bond issuer can make decisions when to call and whether to declare bankruptcy, and the bondholders have the flexibility when to voluntarily convert the bond prior to maturity, the complexity for pricing a callable convertible bond has been considerably amplified. In this section we first construct a structural model to price a non-callable convertible bond, and then decide the optimal voluntary conversion strategy for the bondholders and the optimal bankruptcy strategy for the bond issuer. Using the same methodology, we subsequently derive the analytic valuation of a call-forcing convertible bond and determine the optimal call and bankruptcy strategies for the bond issuer. Based on the assumption that there is no interaction between the optimal voluntary conversion strategy and the optimal call policy, we ultimately obtain the analytic valuation of a callable convertible bond subject to the default risk of the issuer.

2.1. Pricing a non-callable convertible bond with default risk

For a non-callable convertible bond, the bond issuer can decide when to go bankrupt and the bondholders can determine when to voluntarily convert the bonds into common shares. Leland and Toft (1996) use a (single) barrier option approach to valuing a risky corporate coupon bond, motivating us to use a double-barrier option approach to pricing a risky non-callable convertible bond. Similar to Leland and Toft (1996), the initial lower barrier, V_{B1}^{0}, represents the bankruptcy trigger of the issuer. Further we denote the initial upper barrier, V_{Con}^{0}, as the voluntary conversion trigger of the bondholders. We initially treat these two barriers as two exogenous constants, which will be endogenously determined by the Nash-equilibrium argument later.

Under our risk-neutral framework, the initial value of a non-callable convertible bond, $NCCB(0)$, can be written by

$$NCCB(0) = \mathrm{E}^{Q}\left[e^{-r\tau_{B1}^{0}} 1_{\left\{\tau_{B1}^{0}<\tau_{Con}^{0},\tau_{B1}^{0}\le T\right\}} (1-\alpha)V_{B1}^{0} \right] + \mathrm{E}^{Q}\left[e^{-r\tau_{Con}^{0}} 1_{\left\{\tau_{Con}^{0}<\tau_{B1}^{0},\tau_{Con}^{0}\le T\right\}} \gamma V_{Con}^{0} \right]$$

$$+\mathrm{E}^{Q}\left[e^{-rT} 1_{\left\{\min(\tau_{Con}^{0},\tau_{B1}^{0})>T\right\}} P \right] + \mathrm{E}^{Q}\left[\int_{0}^{\tau_{B1}^{0}\wedge\tau_{Con}^{0}\wedge T} Ce^{-rt}dt \right], \tag{2}$$

where $1_{\{A\}}$ denotes the indicator function with value 1 if event A occurs and with value zero otherwise, $t \wedge s \equiv \min(t,s)$, and $\tau_{B1}^{0} \equiv \inf\left(t>0: V(t) \le V_{B1}^{0}\right)$ and $\tau_{Con}^{0} \equiv \inf\left(t>0: V(t) \ge V_{Con}^{0}\right)$ stand for the time of the bankruptcy of the bond issuer and the time of the voluntary conversion of the bondholders, respectively. For the right hand side of Equation (2), the first term denotes the discounted recovery value of a non-callable convertible bond when the bankruptcy occurs prior to the voluntary conversion and to the maturity of the bond. Next, the second term represents the discounted voluntary conversion value when the block conversion happens before the bankruptcy and the maturity. The third term shows the discounted par value when there is no occurrence of the bankruptcy and the voluntary conversion prior to the maturity, and the last term designates the discounted value of the cumulative coupon payments which may be truncated either by the voluntary conversion or by the bankruptcy.

We can then rearrange Equation (2) as follows:

$$NCCB(0) = \frac{C}{r} + \mathrm{E}^{Q}\left[e^{-r\tau_{B1}^{0}} 1_{\left\{\tau_{B1}^{0}<\tau_{Con}^{0},\tau_{B1}^{0}\le T\right\}} \left((1-\alpha)V_{B1}^{0} - \frac{C}{r} \right) \right]$$

$$+\mathrm{E}^{Q}\left[e^{-rT} 1_{\left\{\min\left(\tau_{Con}^{0},\tau_{B1}^{0}\right)>T\right\}} \left(P - \frac{C}{r} \right) \right] + \mathrm{E}^{Q}\left[e^{-r\tau_{Con}^{0}} 1_{\left\{\tau_{Con}^{0}<\tau_{B1}^{0},\tau_{Con}^{0}\le T\right\}} \left(\gamma V_{Con}^{0} - \frac{C}{r} \right) \right]. \tag{3}$$

Since V_{B1}^0 and V_{Con}^0 are two constants at the initial time, Equation (3) can be further simplified as follows:

$$NCCB(0) = \frac{C}{r} + \left((1-\alpha)V_{B1}^0 - \frac{C}{r} \right) G_{\tau_{B1}^0} + \left(P - \frac{C}{r} \right) G_T + \left(\gamma V_{Con}^0 - \frac{C}{r} \right) G_{\tau_{Con}^0}, \tag{4}$$

where $G_{\tau_{B1}^0} \equiv \mathrm{E}^Q\left[e^{-r\tau_{B1}^0} 1_{\left\{\tau_{B1}^0 < \tau_{Con}^0, \tau_{B1}^0 \le T\right\}} \right]$, $G_T \equiv \mathrm{E}^Q\left[e^{-rT} 1_{\left\{\min\left(\tau_{Con}^0, \tau_{B1}^0\right) > T\right\}} \right]$, and $G_{\tau_{Con}^0} \equiv \mathrm{E}^Q\left[e^{-r\tau_{Con}^0} 1_{\left\{\tau_{Con}^0 < \tau_{B1}^0, \tau_{Con}^0 \le T\right\}} \right]$. Here $\mathrm{E}^Q\left[1_{\left\{\tau_{B1}^0 < \tau_{Con}^0, \tau_{B1}^0 \le T\right\}} \right]$ is the risk-neutral probability of the bankruptcy which happens before the voluntary conversion and the maturity of the bond; similarly, $\mathrm{E}^Q\left[1_{\left\{\tau_{Con}^0 < \tau_{B1}^0, \tau_{Con}^0 \le T\right\}} \right]$ is also the risk-neutral probability of the voluntary conversion which occurs prior to the time of the bankruptcy and the maturity of the bond; $\mathrm{E}^Q\left[1_{\left\{\min\left(\tau_{Con}^0, \tau_{B1}^0\right) > T\right\}} \right]$ can be regarded as the risk-neutral probability of the event that the bond survives throughout its remaining life.

In what follows, we take the initial tax benefits of future coupon payments, $TB(0)$, and the initial value of potential bankruptcy costs, $BC(0)$, as two contingent claims upon the unlevered asset value of the firm. By risk-neutral valuation method, the cumulative discounted tax benefits at the initial time can be represented by

$$TB(0) = \mathrm{E}^Q\left[\int_0^{\tau_{B1}^0 \wedge \tau_{Con}^0 \wedge T} \tau C e^{-rt} dt \right] = \mathrm{E}^Q\left[\frac{\tau C}{r}\left(1 - e^{-r(\tau_{B1}^0 \wedge \tau_{Con}^0 \wedge T)} \right) \right] = \frac{\tau C}{r}\left[1 - G_{\tau_{B1}^0} - G_{\tau_{Con}^0} - G_T \right], \tag{5}$$

where τ is the constant corporate tax rate. In view of Equation (5), the tax benefits are cumulated up from the initial time to the maturity of the bond, which may be truncated either by the bankruptcy or by the voluntary conversion. Similarly, the discounted bankruptcy costs at the initial time can be written as

$$BC(0) = \mathrm{E}^Q\left[e^{-r\tau_{B1}^0}\alpha V\left(\tau_{B1}^0\right)1_{\left\{0<\tau_{B1}^0\le T\right\}}\right] = \alpha V_{B1}^0 \mathrm{E}^Q\left[e^{-r\tau_{B1}^0}1_{\left\{0<\tau_{B1}^0\le T\right\}}\right] = \alpha V_{B1}^0 F_{\tau_{B1}^0}, \tag{6}$$

where $F_{\tau_{B1}^0} \equiv \mathrm{E}^Q\left[e^{-r\tau_{B1}^0}1_{\left\{0<\tau_{B1}^0\le T\right\}}\right]$. In particular, $\mathrm{E}^Q\left[1_{\left\{0<\tau_{B1}^0\le T\right\}}\right]$ is the risk-neutral probability that the bond issuer declares bankruptcy before the maturity. Although the bankruptcy costs seem to be irrelevant to the voluntary conversion strategy, we will show later, in fact, that the optimal bankruptcy strategy and the optimal voluntary conversion strategy are mutually interacted.

Consequently, the initial total firm value, $F_{NCCB}(0)$, is equal to the initial unlevered asset value plus the initial tax benefits and less the initial value of the potential bankruptcy costs, i.e., $F_{NCCB}(0) = V(0) + TB(0) - BC(0)$. Since the accounting identity of the balance sheet states that the total firm value must equal to the sum of the equity value and the liability value, the initial equity value of the bond issuer, $E_{NCCB}(0)$, must equal to the initial total firm value minus the initial value of the non-callable convertible bond, i.e., $E_{NCCB}(0) = F_{NCCB}(0) - NCCB(0)$. In order to complete the analytic pricing formulas for $NCCB(0)$, $F_{NCCB}(0)$ and $E_{NCCB}(0)$, the explicit expressions for $G_{\tau_{B1}^0}$, $G_{\tau_{Con}^0}$, G_T, and $F_{\tau_{B1}^0}$ are provided in Appendix.

To endogenize the optimal voluntary conversion policy, V_{Con}^*, and the optimal bankruptcy strategy, V_{B1}^*, we first apply the following smooth-pasting conditions to determine the initial constant voluntary conversion trigger, $V_{Con}^{*,0}$, and the initial constant bankruptcy trigger, $V_{B1}^{*,0}$.

$$\left.\frac{\partial E_{NCCB}(0)}{\partial V(0)}\right|_{V(0)=V_{B1}^{*,0}} = \frac{\partial E_{NCCB}(0)\,|_{V(0)=V_{B1}^{*,0}}}{\partial V_{B1}^{*,0}} = 0, \tag{7}$$

$$\left.\frac{\partial NCCB(0)}{\partial V(0)}\right|_{V(0)=V_{Con}^{*,0}} = \frac{\partial NCCB(0)\,|_{V(0)=V_{Con}^{*,0}}}{\partial V_{Con}^{*,0}} = \gamma . \qquad (8)$$

Equation (7) equates the partial derivative of $E_{NCCB}(0)$ with respect to $V(0)$ evaluated at $V_{B1}^{*,0}$ and the partial derivative of $E_{NCCB}(0)$ evaluated at $V_{B1}^{*,0}$ with respect to $V_{B1}^{*,0}$, which equals to zero. Equation (8) equates the partial derivative of $NCCB(0)$ with respect to $V(0)$ evaluated at $V_{Con}^{*,0}$ and the partial derivative of $NCCB(0)$ evaluated at $V_{Con}^{*,0}$ with respect to $V_{Con}^{*,0}$, which equals to γ. These two conditions represent that at the initial time, the shareholders choose $V_{B1}^{*,0}$ to maximize the equity value, and the bondholders determine $V_{Con}^{*,0}$ to maximize the value of the non-callable convertible bond, respectively.[3] Furthermore, the Nash-equilibrium argument is employed to endogenously determine the optimal strategies for the voluntary conversion and the bankruptcy. Given any $V_{Con}^{*,0}$, the shareholders determine the optimal bankruptcy strategy as a function of $V_{Con}^{*,0}$, denoted as $V_{B1}^{*,0}\left(V_{Con}^{*,0}\right)$; on the other hand, given any $V_{B1}^{*,0}$, the bondholders also decide the optimal conversion strategy as a function of $V_{B1}^{*,0}$, denoted as $V_{Con}^{*,0}\left(V_{B1}^{*,0}\right)$. Under the assumption that both the shareholders and the bondholders are fully informed, the optimal (Nash-equilibrium) strategies for the voluntary conversion and the bankruptcy can be obtained by jointly solving Equations (7) and (8) numerically.

In the previous analyses, we have assumed that both the voluntary conversion trigger and the bankruptcy trigger are constants, which is used to obtain the desired densities and then derive the analytic formula of the non-callable convertible bond. To be consistent with this assumption, we further suppose that all the bondholders and the bond issuer initially regard the optimal voluntary conversion strategy and the optimal bankruptcy strategy, $V_{Con}^{*,0}$ and $V_{B1}^{*,0}$, as two constants; at the next instantaneous time, the bondholders and the bond issuer, who will renew the optimal voluntary conversion and bankruptcy strategies, again take these optimal strategies as the constants that will not vary with time. Repeatedly, the optimal strategies will be updated at every instantaneous time before the maturity given that both the voluntary conversion and the bankruptcy do not occur. After collecting all the optimal strategies, we will observe that these optimal strategies are exactly the solutions of Equations (7) and (8), denoted as $V_{Con}^{*}(0)$ and $V_{B1}^{*}(0)$, which are allowed to change with the time to maturity. This is due to the property of the smooth-pasting conditions, where the optimal strategies are independent of the unlevered asset value, that is, they do not involve any uncertainty and will only vary with the time to maturity. Finally, putting $V_{Con}^{0}=V_{Con}^{*}(0)$ and $V_{B1}^{0}=V_{B1}^{*}(0)$ back into Equation (4), we finish the derivation of the analytic valuation of a non-callable convertible bond subject to the issuer's default risk.

2.2. Pricing a call-forcing convertible bond with default risk

Consider a call-forcing convertible bond, where the bond issuer can decide when to go bankrupt and when to call the bonds back, and the bondholders, however, can not convert voluntarily. Once the bond issuer announces to call the bonds, the bondholders can, at the same time, choose either to accept and then receive the redemption price, or to involuntarily convert the bond into the common shares. The risk-neutral pricing method implies that the initial value of a call-forcing convertible bond, $CFCB(0)$, can be written by

$$CFCB(0)=\mathrm{E}^{Q}\left[e^{-r\tau_{B2}^{0}}1_{\left\{\tau_{B2}^{0}<\tau_{Call}^{0},\tau_{B2}^{0}\leq T\right\}}(1-\alpha)V_{B2}^{0}\right]+\mathrm{E}^{Q}\left[e^{-rT}1_{\left\{\min\left(\tau_{Call}^{0},\tau_{B2}^{0}\right)>T\right\}}P\right]$$

$$+\mathrm{E}^{Q}\left[e^{-r\tau_{Call}^{0}}1_{\left\{\tau_{Call}^{0}<\tau_{B2}^{0},\tau_{Call}^{0}\le T\right\}}\max\left(\gamma V_{Call}^{0},(1+\beta)P\right)\right]+\mathrm{E}^{Q}\left[\int_{0}^{\tau_{B2}^{0}\wedge\tau_{Call}^{0}\wedge T}Ce^{-rt}dt\right], \quad (9)$$

where $\tau_{B2}^{0}\equiv\inf\left(t>0:V(t)\le V_{B2}^{0}\right)$ and $\tau_{Call}^{0}\equiv\inf\left(t>0:V(t)\ge V_{Call}^{0}\right)$ stand for the time of the bankruptcy and the time of the call, respectively. For the right hand side of Equation (9), the first term denotes the discounted recovery value of a call-forcing convertible bond when the bankruptcy occurs prior to the time of the call and the maturity of the bond. The second term represents the discounted par value when there are no call and bankruptcy before the maturity. Next, the third term is the discounted payoff at the time of the call, where the payoff is equal to the maximum amount between the forced conversion value, γV_{Call}^{0}, and the redemption value of the call, $(1+\beta)P$. The last term is the discounted value of the cumulative coupon payments which may be truncated either by the call or by the bankruptcy of the bond issuer.

Since V_{B2}^{0} and V_{Call}^{0} are assumed to be two constants initially, we can also simplify Equation (9) as follows:

$$CFCB(0)=\frac{C}{r}+\left((1-\alpha)V_{B2}^{0}-\frac{C}{r}\right)H_{\tau_{B2}^{0}}+\left(P-\frac{C}{r}\right)H_{T}$$

$$+\left(\max\left(\gamma V_{Call}^{0},(1+\beta)P\right)-\frac{C}{r}\right)H_{\tau_{Call}^{0}}, \quad (10)$$

where $H_{\tau_{B2}^{0}}\equiv\mathrm{E}^{Q}\left[e^{-r\tau_{B2}^{0}}1_{\left\{\tau_{B2}^{0}<\tau_{Call}^{0},\tau_{B2}^{0}\le T\right\}}\right]$, $H_{T}\equiv\mathrm{E}^{Q}\left[e^{-rT}1_{\left\{\min\left(\tau_{Call}^{0},\tau_{B2}^{0}\right)>T\right\}}\right]$, and $H_{\tau_{Call}^{0}}$ $\equiv\mathrm{E}^{Q}\left[e^{-r\tau_{Call}^{0}}1_{\left\{\tau_{Call}^{0}<\tau_{B2}^{0},\tau_{Call}^{0}\le T\right\}}\right]$. Similar to the case of the non-callable convertible bond, various risk-neutral probabilities can be calculated in this case. The total firm value in the call-forcing convertible bond case can also be expressed as

$$F_{CFCB}(0)=V(0)+\frac{\tau C}{r}\left[1-H_{\tau_{B2}^{0}}-H_{\tau_{Call}^{0}}-H_{T}\right]+\tau\beta P1_{\left\{\gamma V_{Call}^{0}<(1+\beta)P\right\}}F_{\tau_{Call}^{0}}-\alpha V_{B2}^{0}F_{\tau_{B2}^{0}}, \quad (11)$$

where $F_{\tau_{B2}^0} \equiv \mathrm{E}^Q\left[e^{-r\tau_{B2}^0} 1_{\left\{0<\tau_{B2}^0 \le T\right\}}\right]$ and $F_{\tau_{Call}^0} \equiv \mathrm{E}^Q\left[e^{-r\tau_{Call}^0} 1_{\left\{0<\tau_{Call}^0 \le T\right\}}\right]$. For the right hand side of Equation (11), the first term is the initial unlevered asset value, and the second term represents the cumulative discounted tax benefits of the coupon payments which may be truncated either by the call or by the bankruptcy. Next, the third term stands for the additional discounted tax benefits of the call premium when the bond is redeemed for cash. The last term expresses the corresponding discounted bankruptcy costs or restructuring costs. Once again, according to the accounting identity of the balance sheet, the initial equity value in this case, $E_{CFCB}(0)$, is equal to the total firm value minus the value of the call-forcing convertible bond. Moreover, the analytic expressions for $H_{\tau_{B2}^0}$, H_T, $H_{\tau_{Call}^0}$, $F_{\tau_{B2}^0}$, and $F_{\tau_{Call}^0}$ are also given in Appendix.

We are now going to determine the optimal call and bankruptcy strategies for the bond issuer who initially chooses these optimal policies by the corresponding smooth-pasting conditions given below.

$$\left.\frac{\partial E_{CFCB}(0)}{\partial V(0)}\right|_{V(0)=V_{B2}^{*,0}} = \frac{\partial E_{CFCB}(0)\,|_{V(0)=V_{B2}^{*,0}}}{\partial V_{B2}^{*,0}} = 0, \tag{12}$$

$$\left.\frac{\partial E_{CFCB}(0)}{\partial V(0)}\right|_{V(0)=V_{Call}^{*,0}} = \frac{\partial E_{CFCB}(0)\,|_{V(0)=V_{Call}^{*,0}}}{\partial V_{Call}^{*,0}} = \begin{cases} 1-\gamma, & \text{if } \gamma V_{Call}^{*,0} \ge (1+\beta)P. \\ 1, & \text{if } \gamma V_{Call}^{*,0} < (1+\beta)P. \end{cases} \tag{13}$$

Similar to Equation (7), Equation (12) equates the partial derivative of $E_{CFCB}(0)$ with respect to $V(0)$ evaluated at $V_{B2}^{*,0}$ and the partial derivative of $E_{CFCB}(0)$ evaluated at $V_{B2}^{*,0}$ with respect to $V_{B2}^{*,0}$, which equals to zero; Equation (13) equates the partial derivative of $E_{CFCB}(0)$ with respect to $V(0)$ evaluated at $V_{Call}^{*,0}$ and the partial derivative of $E_{CFCB}(0)$ evaluated at $V_{Call}^{*,0}$ with respect to $V_{Call}^{*,0}$, which is either equal to $(1-\gamma)$ when the forced conversion occurs, or equal to 1 when the call redemption happens.[4] Furthermore, Equations (12) and (13) represent that the shareholders make decisions on the optimal call and bankruptcy strategies to maximize the equity value. As noted in Sarkar (2003), the shareholders must choose the optimal call policy to maximize the equity value rather than to minimize the value of convertible bonds (such as Ingersoll (1977a) and Brennan and Schwartz (1977)). These two objectives are equivalent in a perfect capital market, but in a market with frictions minimizing the convertible bond value does not imply maximizing the equity value.

Due to the same inference from the previous section, jointly solving $V_{Call}^{*}(0)$ and $V_{B2}^{*}(0)$ from Equations (12) and (13) and then substituting them back into Equation (10), we finally complete the analytic valuation of a call-forcing convertible bond with consideration of the issuer's default risk.

2.3. Pricing a callable convertible bond with default risk

To price a callable convertible bond, we have to determine its optimal strategies for the call, the voluntary conversion, and the bankruptcy. Regarding the optimal call policy and the optimal voluntary conversion strategy of the callable convertible bond, Ingersoll (1977a) has ever shown that whenever it is optimal to voluntarily convert a non-callable convertible bond, it will also be optimal to convert a callable convertible bond since the latter is always no more valuable than the former. We further assume that the possibility of a voluntary conversion (a call) does not affect the optimal call policy (the optimal voluntary conversion strategy). This assumption, which ensures the uncorrelation between the optimal voluntary conversion strategy and the optimal call policy of the callable convertible bond, may be less inappropriate but is used to keep the model tractable. Moreover, we assume that the optimal bankruptcy trigger of the call-forcing convertible bond is that of the callable convertible bond which is otherwise identical. Therefore, we can conclude that the optimal voluntary conversion trigger of the non-callable convertible bond, and the optimal call trigger and the optimal bankruptcy trigger of the call-forcing convertible bond are employed to the callable convertible bond, where all three bonds are otherwise the same.

The intuition is as follows. Recall that the optimal voluntary conversion trigger of the non-callable convertible bond, $V_{Con}^{*}(0)$, is chosen to maximize the value of the non-callable convertible bond, $NCCB(0)$, while the optimal bankruptcy trigger, $V_{B1}^{*}(0)$, is chosen to maximize the corresponding equity value. On the other hand, the optimal bankruptcy trigger and the optimal call trigger of the call-forcing convertible bond, $V_{B2}^{*}(0)$ and $V_{Call}^{*}(0)$, are chosen to maximize the associated equity value, implying to minimize the value of the call-forcing convertible bond, $CFCB(0)$. In addition, the value of a callable convertible bond, $CCB(0)$, is between the values of the call-forcing convertible bond and the non-callable convertible bond, where these three bonds are otherwise identical, namely, $NCCB(0) \geq CCB(0) \geq CFCB(0)$. This is because the more rights the bondholders/shareholders get, the greater premium they need to pay. Due to the assumption that there is no interaction between the optimal voluntary conversion strategy and the optimal call policy for a callable convertible bond, we can therefore conclude that the callable convertible bondholders will choose the same optimal voluntary conversion strategy as $V_{Con}^{*}(0)$ to maximize $CCB(0)$ where $V_{Con}^{*}(0)$ originally maximizes (the more valuable) $NCCB(0)$; similarly, the corresponding equityholders will choose $V_{B2}^{*}(0)$ and $V_{Call}^{*}(0)$ to minimize $CCB(0)$ where $V_{B2}^{*}(0)$ and $V_{Call}^{*}(0)$ originally minimizes (the less valuable) $CFCB(0)$.

Recall that in our model the bond issuer would announce the call policy to all the outstanding callable convertible bonds, and in addition, all the bondholders would voluntarily convert the convertibles at the same time (i.e., block conversion). As a consequence, we have to decide which may occur, either the call or the voluntary conversion. If the optimal call trigger is greater than or equal to the optimal voluntary conversion trigger, then the voluntary conversion may happen while the call will not. That is to say, we can conclude that the analytic valuation of the callable convertible bond is exactly that of the non-callable convertible bond which is otherwise identical. Similarly, if the optimal call trigger is less than the optimal voluntary conversion trigger, then the analytic valuation of the callable convertible bond is just that of the call-forcing convertible bond which is otherwise identical. Hence, the analytic valuation of a callable convertible bond subject to the default risk of the bond issuer can be expressed as follows:

$$CCB\left(0;V_{B2}^{*}(0),V_{Con}^{*}(0),V_{Call}^{*}(0)\right)=\begin{cases} NCCB\left(0;V_{B2}^{*}(0),V_{Con}^{*}(0)\right), \text{ if } V_{Con}^{*}(0)\le V_{Call}^{*}(0). \\ CFCB\left(0;V_{B2}^{*}(0),V_{Call}^{*}(0)\right), \text{ if } V_{Con}^{*}(0)> V_{Call}^{*}(0). \end{cases} \tag{14}$$

Finally, it should be emphasized that the optimal strategies for call, voluntary conversion, and bankruptcy are all time-dependent and are endogenously determined in our model. In addition, although some of the formulas are not directly related to one of the strategies, they may be indirectly affected by the joint determination of the optimal strategies. For example, in Section 2.2, the bankruptcy costs in the case of the call-forcing convertible bond do not directly depend on the optimal call policy but will be indirectly influenced through the simultaneous resolution of the smooth-pasting conditions. Next, the time-dependent optimal strategies in the finite maturity setting are matter-of-fact in comparison with the constant optimal strategies in the perpetual maturity setting of Sarkar (2003) because the optimal strategies do correlate with the time to maturity of a callable convertible bond in reality. Last but not least, the simultaneous endogenous resolution of the optimal strategies, through the rationales of the shareholders and the bondholders, has great insights on the valuation of the callable convertible bond.

3. Numerical Examples

In this section, we implement some numerical examples to characterize the optimal strategies for call, voluntary conversion, and bankruptcy as well as the valuation of a callable convertible bond. In particular, we examine the effects on the optimal strategies and on the bond values with respect to the bond's maturity, the coupon payment, the return volatility of the unlevered asset, and the risk-free interest rate.

3.1. Optimal strategies for call and voluntary conversion

The parameters in the base case, taken from Sarkar (2003), are as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\sigma = 0.2$, $\beta = 0.05$, $\gamma = 0.2$, and $T = 5$.[5] All parameters in this article are the same as the base case unless otherwise stated. Also notice that in the numerical analyses of this article, the desired pricing formula of the callable convertible bond, involving some infinite sums (from zero to infinity), has been replaced with the finite sums (assumed from zero to ten).[6]

Figure 1 illustrates the optimal call triggers as a function of the time to maturity for various return volatilities of the unlevered asset. The optimal call trigger is an increasing function of the time to maturity when the time to maturity becomes shorter, and is a decreasing function of the time to maturity otherwise, that is, the optimal call trigger is concave to the time to maturity. The concavity is more obvious as the return volatility becomes larger, and therefore, the riskier bond issuer will call back the bond at higher unlevered asset value, which results in late calls. Figure 2 shows the relationship between the optimal call triggers and the risk-free interest rate for varying coupon payments. Higher coupon payments will raise the optimal call trigger, which is convex to the risk-free interest rate except for the case of zero coupon payment. The reason may be that the present value of future potential tax benefits is greater for the bond issuer as the coupon payment increases and the risk-free interest rate decreases, and hence, the optimal call trigger is relatively high and the possibility of the call becomes extremely small. As for the case of zero coupon payment, the optimal call trigger is insensitive to the risk-free interest rate due to the absence of tax benefits.

Many callable convertible bonds are called too late relative to the optimal call policy derived by Ingersoll (1977a), which equals to $\frac{1}{\gamma}(1+\beta)P$. In our base case, it is equal to 525, and hence, a late call in our model means that the optimal call trigger is greater than this amount. Ederington et al. (1997) provide some empirical evidences that the rule of Ingersoll (1977a) has not been followed in most of the time, and the issuers of callable convertible bonds have often ignored the opportunities to force the bond investors to exercise their American conversion options. They also discover that the issuers delay exercising their call options so as to maintain the value of tax benefits embedded in the convertible bonds, and that they are more likely to delay calls to the extent that the underlying stock is paying dividends. In our pricing model, the effects of tax benefits and dividends are taken into consideration, and the numerical results of Figures 1 and 2 show that late calls exist in most of the cases. In particular, higher coupon payment, lower risk-free interest rate, greater return volatility, and medium time to maturity will cause late calls where the optimal call trigger is extraordinarily high, which is generally consistent with Ederington et al. (1997).

The optimal voluntary conversion triggers, plotted in Figures 3 and 4, behave much similarly to the optimal call triggers in Figures 1 and 2, respectively. Some implications of our model are discussed as follows. The optimal voluntary conversion triggers are usually greater than the optimal call triggers for the most part, that is, the voluntary conversion will not occur in most of the cases. Nevertheless, our model predicts that for a callable convertible bond with very low coupon payments or with shorter time to maturity, smaller return volatility, and higher risk-free interest rate, the voluntary conversion may happen. Moreover, the numerical results also confirm that when the call premium is low enough, the call redemption of a callable convertible bond may not take place. On the other hand, for a callable convertible bond with higher call premium, higher coupon payment, shorter time to maturity, smaller return volatility and the intermediate risk-free interest rate, the call redemption may occur.

3.2. Optimal bankruptcy strategy

Figure 5 plots the optimal bankruptcy trigger as a function of the time to maturity for various return volatilities of the unlevered asset. Observe that the optimal bankruptcy trigger is a decreasing function of the time to maturity and is concave to the time to maturity. In addition, similar to Leland and Toft (1996), the greater the return volatility, the lower the optimal bankruptcy trigger due to the limited liability of equityholders. In Figure 6, generally speaking, rising coupon payments will increase the optimal bankruptcy trigger, which is a decreasing function of the risk-free interest rate, and this result is consistent with Leland (1994). The reason may be that the effect of the present value of future potential coupon payments dominates the effect of the present value of future potential tax benefits. Nevertheless, the behavior of the optimal bankruptcy trigger is reversed while the risk-free interest rate is rather low since the effect of the tax benefits is dominant in this case. For the case of zero coupon payment, which stands for the pure effect of the risk-free interest rate, the optimal bankruptcy trigger is slightly convex to and is a decreasing function of the risk-free interest rate.

3.3. Values of the callable convertible bond

Figure 7 shows the values of a callable convertible bond as a joint function of the unlevered asset value and the time to maturity. The value of a callable convertible bond is a non-decreasing function of both the unlevered asset value and the time to maturity. For lower unlevered asset value, the callable convertible bond value, concave to the unlevered asset value, is analogous to the price of a risky coupon bond because the possibilities of the call and the voluntary conversion are extremely small. On the other hand, for higher unlevered asset value, the callable convertible bond value, convex to the unlevered asset value, is similar to the equity value due to increases in the possibilities of the call and the voluntary conversion. Moreover, when the time to maturity is short and the unlevered asset value is in the middle range, the callable convertible bond value, similar to the risk-free coupon bond value, is very close to the par value, which equals to 100 in our base case. This is because the events of call, voluntary conversion, and bankruptcy scarcely happen in this case.

Figure 8 illustrates the prices of a callable convertible bond as a function of the unlevered asset value for various return volatilities. Not only will greater return volatilities increase the probability of the bankruptcy but also will raise the probabilities of the call and the voluntary conversion. For lower unlevered asset value, the former effect is dominant and thus the callable convertible bond value decreases as the return volatility goes up. However, for higher unlevered asset value, the latter effect dominates and hence, the callable convertible bond value increases with rising return volatilities. In addition, related to our earlier discussion in Figure 7, we observe that higher and lower unlevered asset values make the callable convertible bond behave like the equity and the risky coupon bond, respectively. Therefore, under higher unlevered asset value, an increase in the volatility can raise the price of a callable convertible bond due to the property of the equity value. On the other hand, the callable convertible bond acts as the risky coupon bond under lower unlevered asset value and consequently, the higher the return volatility, the lower the price of a callable convertible bond.

Table 1 exhibits the values of the callable convertible bond for varying coupon payments, risk-free interest rates, and unlevered asset values. An increase in the coupon payment can raise the value of the callable convertible bond in most of the cases, which accords well with the intuition. However, there are some significant exceptions when the unlevered asset is 700, the coupon payment equals to 0, 1, and 3 among all various risk-free interest rates (excluding the case of $V = 700$, $C = 3$ and $r = 0.01$). In view of Figure 2, we can observe that in these exceptional cases, the optimal call triggers are less than 700, and thus the callable convertible bond has been called back and will be forced to convert into common shares. As a result, it is similar to the equity whose value falls as the coupon payment increases.

Higher risk-free interest rate causes lower values of the callable convertible bond for the most part, corresponding with the general property of bonds. Nevertheless, there are three sorts of significant anomalies in Table 1. First of all, increasing risk-free interest rate raises the values of the callable convertible bond when $V = 100$, $C = 0$ with $r = 0.01$ and $r = 0.03$, and when $V = 100$, $C = 9$ with $r = 0.05$ and $r = 0.07$. This anomaly can be explained by Figure 6, which shows the above two cases correspond to higher optimal bankruptcy triggers and thus suffer from more bankruptcy risk. The callable convertible bond is, therefore, analogous to a kind of junk bonds, of which the price is positively correlated to the risk-free interest rate, as noted in Leland (1994). Next, the second anomaly emerges as $V = 700$, $C = 1$ with

$r = 0.01$ and $r = 0.03$. According to our earlier analyses, the callable convertible bond has been converted into the common shares in this case, and hence the bond price, similar to the equity value, increases with a rise in the risk-free interest rate. Finally, the last anomaly appears when $V = 700$, $C = 7$ with $r = 0.01$ and $r = 0.03$, and when $V = 700$, $C = 9$ with $r = 0.01$ and $r = 0.03$. From Figures 2 and 6, when $r = 0.01$ with $C = 7$ and $C = 9$, the optimal call triggers are extremely high compared with the unlevered asset value, 700, and the optimal bankruptcy triggers are almost close to zero. As a result, the callable convertible bond is nearly similar to a default-free coupon bond. However, when the risk-free interest rate increases from 0.01 to 0.03, the optimal call triggers sharply decrease and the optimal bankruptcy triggers slightly rise. Hence, the reason of the last anomaly is the effect of increasing call probabilities dominates the effect of increasing bankruptcy probabilities. In addition, Table 1 also shows that the values of a callable convertible bond, characterized with higher coupon payments and lower risk-free interest rate, are more sensitive to the risk-free interest rate, especially in the case of higher unlevered asset value.

4. Asset Substitution Problem

The asset substitution/risk shifting problem states that shareholders wish to increase the riskiness of firm's activities so as to transfer value from bondholders to themselves. For example, shareholders may adopt a riskier investment project with negative net present value (NPV). Some structural models, such as Merton (1974), explicitly regard the equity value as a call option on the firm's asset value, and therefore, the asset substitution problem appears in such models due to the Vega property of the call option, that is, increasing the return volatility of the firm's asset will result in higher equity values. Barnea et al. (1980) explore this analogy and suggest that issuing shorter-term debts may reduce the incentives of shareholders to increase risk. In addition, the monitoring role of convertible debts in resolving risk shifting problems is studied in the literatures. For instance, Green (1984) shows that convertible bonds can be used to restore the positive NPV maximization rule of the shareholders. Recently, Chesney and Gibson-Asner (2001) propose a down-and-out equity valuation model that applies to a leveraged firm facing the asset-based exogenous solvency rule, and they also show that the optimal volatility selection of shareholders is lower in the case where the firms financed with convertible debts. The equity in our model, however, can not be viewed as a simple down-and-out barrier option. This is because in our setting, default can occur at any time during the life of the debt and the optimal bankruptcy trigger also varies with the riskiness of the firm's activities; most importantly, tax benefits and bankruptcy costs are taken into consideration to derive the equity value.

To clearly illustrate whether issuing convertible bonds instead of coupon bonds can reduce the risk shifting problem, we first consider a risky coupon bond as a sole debt obligation in our framework. Following the same methodology in Section 2, the risky coupon bond price, the total firm value, and the equity value can then be obtained, and the optimal bankruptcy strategy can be also endogenously determined by the corresponding smooth-pasting condition, which is similar to Equation (7) or Equation (12). Moreover, we can define the risk shifting intensity as the partial derivative of the equity value with respect to the return volatility of the unlevered asset. Hence, the positive risk shifting intensity represents that shareholders have incentives to increase the riskiness of firm's activities. Using the parameters of the base case in the previous section, we provide Figure 9 to compare the risk shifting intensities between the coupon-bond-based model (where the coupon bond is the only debt obligation) and the callable-convertible-bond-based model (where the callable convertible bond is the only debt of the firm, i.e., the same model as in Section 2.3).

Figure 9 plots the risk shifting intensities as a function of the unlevered asset value. Panels 1-1 and 1-2 plot the risk shifting intensities of the coupon-bond-based model with the time to maturities of 6 months and 5 years, respectively. Panels 2-1 and 2-2 plot the risk shifting intensities of the callable-convertible-bond-based model with the time to maturities of 6 months and 5 years, respectively. Observe that (i) Panels 1-1 and 1-2 display that the risk shifting intensities approach to zero as the unlevered asset value goes up, that is, there is almost no asset substitution problem when the default risk is rather small; (ii) shorter time to maturities will reduce the asset substitution problem both in the coupon-bond-based model and the callable-convertible-bond-based model, which is generally consistent with Barnea et al. (1980); (iii) the hypothesis that callable convertible bonds can be used to resolve the risk shifting problem is numerically validated by comparing Panels 1-1 and 1-2 with the corresponding Panels 2-1 and 2-2; (iv) Panels 2-1 and 2-2 show that positive risk shifting intensities appear again as the unlevered asset value becomes higher because these callable convertible bonds have been called back and forced to convert into common shares.

5. Concluding Remarks

In this article, we construct a structural model to derive the analytic valuation of a callable convertible bond by the pricing method of double-barrier options with consideration of the possibilities that the call, the voluntary conversion, and the bankruptcy can occur prior to the maturity of the bond. Our model also takes the bankruptcy cost, the tax benefit, and the time to maturity of the bond into account. Not only are the optimal call and bankruptcy strategies endogenously determined by the shareholders as the equity value is maximized, but also the optimal voluntary conversion strategy is obtained by the bondholders while the value of the convertible bond is maximized.

In summary, our numerical results predict that (i) late calls are in most of the cases, and higher coupon, lower risk-free interest rate, greater return volatility, and medium time to maturity will lead to the extreme late call case where the optimal call trigger becomes extraordinarily high, which is generally consistent with Ederington et al. (1997); (ii) the voluntary conversion may occur in the cases of the callable convertible bond with very low coupon payments, or with shorter time to maturity, smaller return volatility, and higher risk-free interest rate; (iii) the call redemption may take place in the case of the callable convertible bond with greater call premium, higher coupon payment, shorter time to maturity, smaller return volatility and the intermediate risk-free interest rate. In addition, our model suggests that shorter-term bonds are useful to reconcile the asset substitution problem, which is consistent with Barnea et al. (1980). Particularly, we have confirmed the hypothesis that callable convertible bonds can be used to reduce the risk shifting problem.

By and large, the analytic valuation of the callable convertible bond in our model is simple, yet complete. Further, traders and portfolio managers of convertible bonds may benefit from the pricing formula and the predictions of the numerical results to decide their trading strategies. In our model, the risk-free interest rate is assumed to be constant, and in addition, the debt structure of the firm is supposed to be a single callable convertible bond. In regard to these simplifications, taking other debt obligations, such as corporate coupon bonds and bonds with warrants, into consideration is a straightforward extension of our model while the more complex capital structures are desired to be determined. Moreover, the analytic valuation of a callable convertible bond with zero coupon payment (similar to Liquid Yield Option Notes, LYONs) under stochastic interest rates can be also directly derived by a time-change Brownian motion technique in our framework.

Appendix[7]

In this Appendix we provide the analytic expressions of $G_{\tau_{B1}^0}$, G_T, $G_{\tau_{Con}^0}$, $F_{\tau_{B1}^0}$, $H_{\tau_{B2}^0}$, H_T, $H_{\tau_{Call}^0}$, $F_{\tau_{B2}^0}$, and $F_{\tau_{Call}^0}$, which can be divided into two parts. One is related to the distribution of a stopping time of a standard Brownian motion with constant drift. The other is associated with the joint distribution of two stopping times of a standard Brownian motion with constant drift.

Regarding the first part, many books which discuss standard Brownian motions would provide the desired distribution, for example, Chapter 1 of Harrison (1985). By Girsanov Theorem and the technique of completing squares, the following formulas can be derived.

$$F_{\tau_{B1}^0} = \left(\frac{V(0)}{V_{B1}^0}\right)^{\frac{\lambda^*-\lambda}{\sigma^2}} \left[\Phi\left(\frac{\ln \frac{V_{B1}^0}{V(0)} - \lambda^* T}{\sigma\sqrt{T}} \right) + \left(\frac{V(0)}{V_{B1}^0}\right)^{\frac{-2\lambda^*}{\sigma^2}} \Phi\left(\frac{\ln \frac{V_{B1}^0}{V(0)} + \lambda^* T}{\sigma\sqrt{T}} \right) \right],$$

$$F_{\tau_{B2}^0} = \left(\frac{V(0)}{V_{B2}^0}\right)^{\frac{\lambda^*-\lambda}{\sigma^2}} \left[\Phi\left(\frac{\ln \frac{V_{B2}^0}{V(0)} - \lambda^* T}{\sigma\sqrt{T}} \right) + \left(\frac{V(0)}{V_{B2}^0}\right)^{\frac{-2\lambda^*}{\sigma^2}} \Phi\left(\frac{\ln \frac{V_{B2}^0}{V(0)} + \lambda^* T}{\sigma\sqrt{T}} \right) \right], \text{ and}$$

$$F_{\tau_{Call}^0} = \left(\frac{V_{Call}^0}{V(0)}\right)^{\frac{\lambda-\lambda^*}{\sigma^2}} \left[\Phi\left(\frac{\ln \frac{V(0)}{V_{Call}^0} + \lambda^* T}{\sigma\sqrt{T}} \right) + \left(\frac{V_{Call}^0}{V}\right)^{\frac{2\lambda^*}{\sigma^2}} \Phi\left(\frac{\ln \frac{V(0)}{V_{Call}^0} - \lambda^* T}{\sigma\sqrt{T}} \right) \right],$$

where $\lambda = r - q - 0.5\sigma^2$, $\lambda^* = \sqrt{\lambda^2 + 2r\sigma^2}$, and $\Phi(\cdot)$ denotes the cumulative standard normal distribution.

As for the second part, we refer to Kolkiewicz (2002) where the desired distributions are provided in a systematic way. Notice that in the following results we implicitly assume that the upper barrier and the lower barrier do not intersect during the remaining life of the bond. Again, Girsanov Theorem and the use of completing squares will result in the following formulas:

$$G_{\tau_{B1}^0}=\left(\frac{V_{B1}^0}{V(0)}\right)^{\frac{\lambda}{\sigma^2}}\sum_{n=0}^{\infty}\left\{\left[\left(\frac{(V_{Con}^0)^{2n}V(0)}{(V_{B1}^0)^{2n+1}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n+1}}{V(0)(V_{Con}^0)^{2n}}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{(V_{B1}^0)^{2n+1}}{(V_{Con}^0)^{2n}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\right.\right.$$

$$\left.\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n+1}}{V(0)(V_{Con}^0)^{2n}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]-\left[\left(\frac{(V_{Con}^0)^{2n+2}}{(V_{B1}^0)^{2n+1}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n+1}V(0)}{(V_{Con}^0)^{2n}}-\lambda^*T}{\sigma\sqrt{T}}\right)\right.$$

$$\left.\left.+\left(\frac{(V_{B1}^0)^{2n+1}V(0)}{(V_{Con}^0)^{2n+2}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n+1}V(0)}{(V_{Con}^0)^{2n+2}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]\right\},$$

$$G_{\tau_{Con}^0}=\left(\frac{V_{Con}^0}{V(0)}\right)^{\frac{\lambda}{\sigma^2}}\sum_{n=0}^{\infty}\left\{\left[\left(\frac{(V_{Con}^0)^{2n+1}}{(V_{B1}^0)^{2n}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n}V(0)}{(V_{Con}^0)^{2n+1}}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{(V_{B1}^0)^{2n}V(0)}{(V_{Con}^0)^{2n+1}}\right)^{\frac{\lambda^*}{\sigma^2}}\right.\right.$$

$$\left.\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n}V(0)}{(V_{Con}^0)^{2n+1}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]-\left[\left(\frac{(V_{Con}^0)^{2n+1}V(0)}{(V_{B1}^0)^{2n+2}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n+2}}{(V_{Con}^0)^{2n+1}V(0)}-\lambda^*T}{\sigma\sqrt{T}}\right)\right.$$

$$\left.\left.+\left(\frac{(V_{B1}^0)^{2n+2}}{(V_{Con}^0)^{2n+1}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n+2}}{(V_{Con}^0)^{2n+1}V(0)}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]\right\},$$

$$G_T=e^{-rT}\sum_{n=-\infty}^{\infty}\left\{\left(\frac{V_{Con}^0}{V_{B1}^0}\right)^{\frac{2n\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n}}{V(0)(V_{Con}^0)^{2n-1}}-\lambda T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{\ln\frac{(V_{B1}^0)^{2n+1}}{V(0)(V_{Con}^0)^{2n}}-\lambda T}{\sigma\sqrt{T}}\right)\right]\right.$$

$$-\left(\frac{(V_{B1}^0)^{n+1}}{(V_{Con}^0)^n V(0)}\right)^{\frac{2\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{(V_{Con}^0)^{2n+1}V(0)}{(V_{B1}^0)^{2n+2}}-\lambda T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{\ln\frac{(V_{Con}^0)^{2n}V(0)}{(V_{B1}^0)^{2n+1}}-\lambda T}{\sigma\sqrt{T}}\right)\right]\Bigg\},$$

$$H_{\tau_{B2}^0}=\left(\frac{V_{B2}^0}{V(0)}\right)^{\frac{\lambda}{\sigma^2}}\sum_{n=0}^{\infty}\Bigg\{\Bigg[\left(\frac{(V_{Call}^0)^{2n}V(0)}{(V_{B2}^0)^{2n+1}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n+1}}{V(0)(V_{Call}^0)^{2n}}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{(V_{B2}^0)^{2n+1}}{(V_{Call}^0)^{2n}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}$$

$$\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n+1}}{V(0)(V_{Call}^0)^{2n}}+\lambda^*T}{\sigma\sqrt{T}}\right)\Bigg]-\Bigg[\left(\frac{(V_{Call}^0)^{2n+2}}{(V_{B2}^0)^{2n+1}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n+1}V(0)}{(V_{Call}^0)^{2n}}-\lambda^*T}{\sigma\sqrt{T}}\right)$$

$$+\left(\frac{(V_{B2}^0)^{2n+1}V(0)}{(V_{Call}^0)^{2n+2}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n+1}V(0)}{(V_{Call}^0)^{2n+2}}+\lambda^*T}{\sigma\sqrt{T}}\right)\Bigg]\Bigg\},$$

$$H_{\tau_{Call}^0}=\left(\frac{V_{Call}^0}{V(0)}\right)^{\frac{\lambda}{\sigma^2}}\sum_{n=0}^{\infty}\Bigg\{\Bigg[\left(\frac{(V_{Call}^0)^{2n+1}}{(V_{B2}^0)^{2n}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n}V(0)}{(V_{Call}^0)^{2n+1}}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{(V_{B2}^0)^{2n}V(0)}{(V_{Call}^0)^{2n+1}}\right)^{\frac{\lambda^*}{\sigma^2}}$$

$$\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n}V(0)}{(V_{Call}^0)^{2n+1}}+\lambda^*T}{\sigma\sqrt{T}}\right)\Bigg]-\Bigg[\left(\frac{(V_{Call}^0)^{2n+1}V(0)}{(V_{B2}^0)^{2n+2}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n+2}}{(V_{Call}^0)^{2n+1}V(0)}-\lambda^*T}{\sigma\sqrt{T}}\right)$$

$$+\left(\frac{(V_{B2}^0)^{2n+2}}{(V_{Call}^0)^{2n+1}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n+2}}{(V_{Call}^0)^{2n+1}V(0)}+\lambda^*T}{\sigma\sqrt{T}}\right)\Bigg]\Bigg\},\text{ and}$$

$$H_T=e^{-rT}\sum_{n=-\infty}^{\infty}\Bigg\{\left(\frac{V_{Call}^0}{V_{B2}^0}\right)^{\frac{2n\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n}}{V(0)(V_{Call}^0)^{2n-1}}-\lambda T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n+1}}{V(0)(V_{Call}^0)^{2n}}-\lambda T}{\sigma\sqrt{T}}\right)\right]$$

$$-\left(\frac{(V_{B2}^0)^{n+1}}{(V_{Call}^0)^n V(0)}\right)^{\frac{2\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{(V_{Call}^0)^{2n+1}V(0)}{(V_{B2}^0)^{2n+2}}-\lambda T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{\ln\frac{(V_{Call}^0)^{2n}V(0)}{(V_{B2}^0)^{2n+1}}-\lambda T}{\sigma\sqrt{T}}\right)\right]\Bigg\}.$$

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Footnotes

[1] As long as a firm remains solvent, it will roll over the matured debts by issuing new debts with the identical provisions.

[2] The assumption of the constant interest rate seems to be a limitation, but Brenann and Schwartz (1980) have ever indicated that the pricing errors are likely to be small.

[3] All of the optimal strategies in the numerical examples of this article have been checked for the second order sufficient condition of maximization.

[4] In views of Equations (10) and (11), the equity value evaluated at $V_{Call}^{*,0}$ is equal to

$$V_{Call}^{*,0} - \left(\max\left(\gamma V_{Call}^{*,0}, (1+\beta)P \right) - \frac{C}{r} \right).$$

[5] Note that the maturity of the bond is infinite in Sarkar (2003).

[6] The similar finite sum from zero to six in Kolkiewicz (2002) will achieve the precision at the 1×10^{-4} level.

[7] In Appendix, all notations are defined in the text. The derivations of the formulas are rather lengthy and are available upon request.

Table 1

Values of the callable convertible bond for various coupon payments C risk-free interest rates r, and unlevered asset values V *

		$C=0$	$C=1$	$C=3$	$C=5$	$C=7$	$C=9$
$V=100$	$r=0.01$	70.1304	79.8774	99.9895	118.831	129.262	139.016
	$r=0.03$	76.0017	78.064	81.3014	83.4202	84.5099	84.6794
	$r=0.05$	73.8349	76.6368	81.1219	84.0518	85.3629	84.9772
	$r=0.07$	69.1157	72.5535	78.6639	83.6004	87.1546	89.0153
	$r=0.09$	63.3819	67.0840	74.0637	80.3516	85.7869	90.0955
$V=400$	$r=0.01$	85.2181	104.093	115.312	121.729	130.018	139.270
	$r=0.03$	85.6840	92.7388	105.112	114.616	122.886	130.972
	$r=0.05$	85.1181	89.3716	97.9861	106.110	113.915	121.681
	$r=0.07$	84.4142	87.4606	93.8555	100.333	106.946	113.800
	$r=0.09$	83.5488	85.9600	91.0112	96.2024	101.657	107.494
$V=700$	$r=0.01$	172.525	136.598	155.878	155.601	150.698	150.474
	$r=0.03$	170.140	146.043	138.438	146.410	155.064	161.722
	$r=0.05$	157.296	145.580	138.539	141.096	146.529	152.405
	$r=0.07$	151.048	143.882	139.065	140.444	144.093	148.402
	$r=0.09$	148.521	142.986	139.323	140.526	143.488	146.918

* All parameters in this table are the same as the base case in the text unless otherwise noted, and the optimal strategies for call, voluntary conversion, and bankruptcy are determined endogenously.

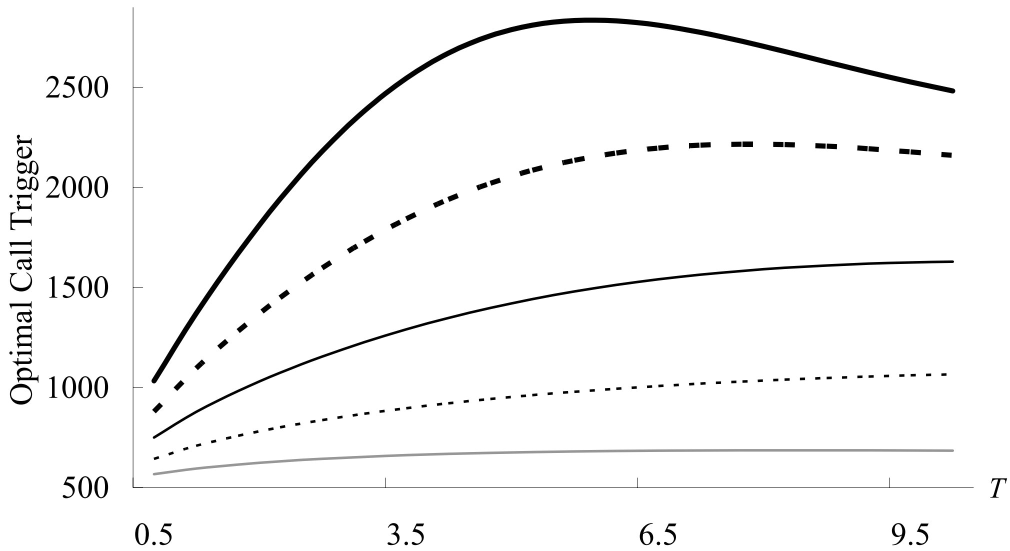

Figure 1

Optimal call triggers as a function of the time to maturity T for various return volatilities σ

The lines plot the optimal call triggers as a function of the time to maturity with return volatilities of 0.1 (gray line), 0.3 (dashed line), 0.5 (solid line), 0.7 (bold dashed line), and 0.9 (bold solid line). The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\beta = 0.05$, and $\gamma = 0.2$.

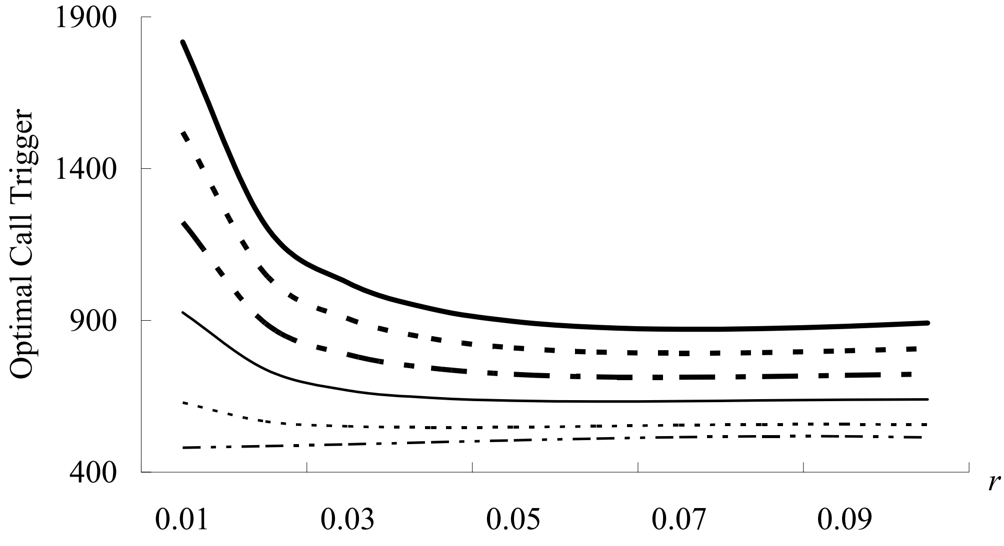

Figure 2

Optimal call triggers as a function of the risk-free interest rate r for various coupon payments C

The lines plot the optimal call triggers as a function of the risk-free interest rate with coupon payments of 0 (solid-dashed line), 1 (dashed line), 3 (solid line), 5 (bold solid-dashed line), 7 (bold dashed line), and 9 (bold solid line). The parameters are given as follows: $P = 100$, $\tau = 0.35$, $\alpha = 0.5$, $q = 0.04$, $\sigma = 0.2$, $\beta = 0.05$, $\gamma = 0.2$, and $T = 5$.

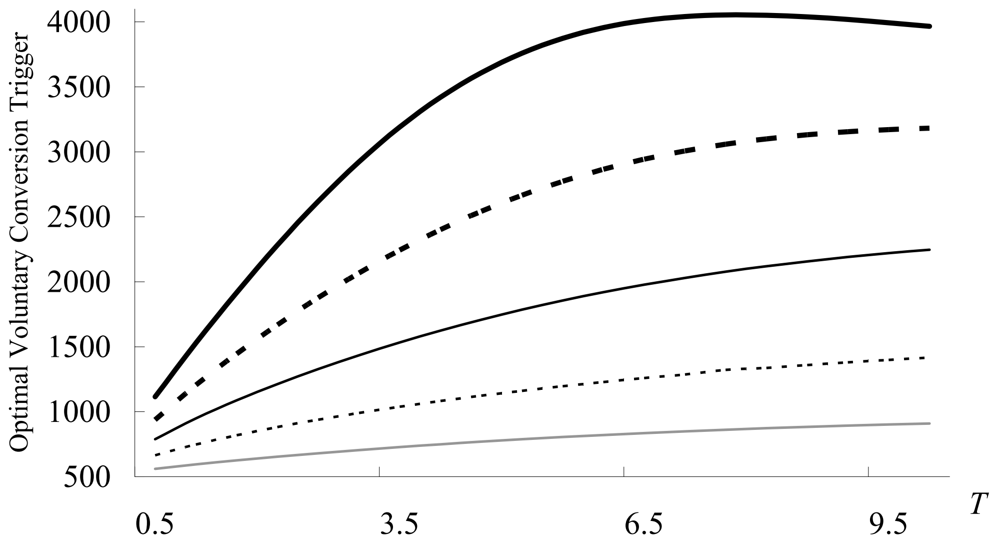

Figure 3

Optimal voluntary conversion triggers as a function of the time to maturity T for various return volatilities σ

The lines plot the optimal voluntary conversion triggers as a function of the time to maturity with return volatilities of 0.1 (gray line), 0.3 (dashed line), 0.5 (solid line), 0.7 (bold dashed line), and 0.9 (bold solid line). The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\beta = 0.05$, and $\gamma = 0.2$.

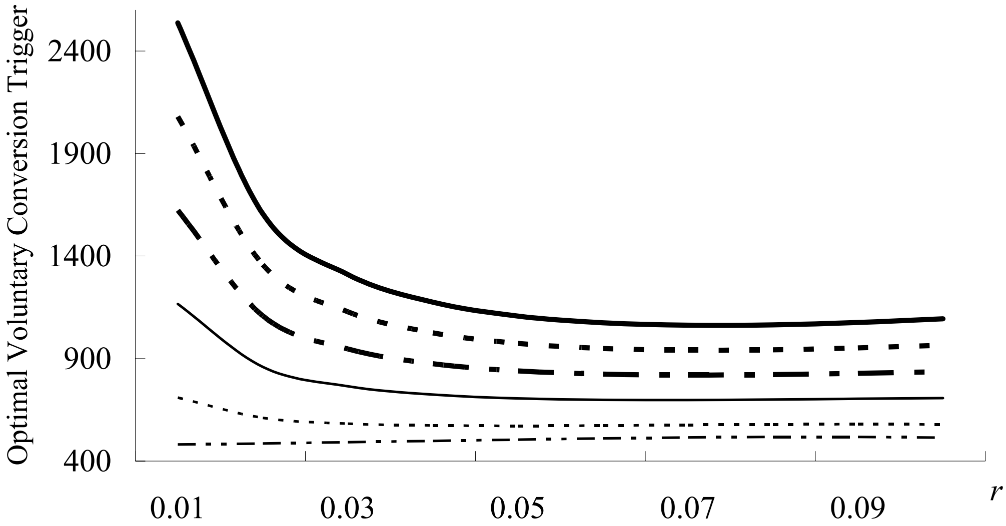

Figure 4

Optimal voluntary conversion triggers as a function of the risk-free interest rate r for various coupon payments C

The lines plot the optimal voluntary conversion triggers as a function of the risk-free interest rate with coupon payments of 0 (solid-dashed line), 1 (dashed line), 3 (solid line), 5 (bold solid-dashed line), 7 (bold dashed line), and 9 (bold solid line). The parameters are given as follows: $P = 100$, $\tau = 0.35$, $\alpha = 0.5$, $q = 0.04$, $\sigma = 0.2$, $\beta = 0.05$, $\gamma = 0.2$, and $T = 5$.

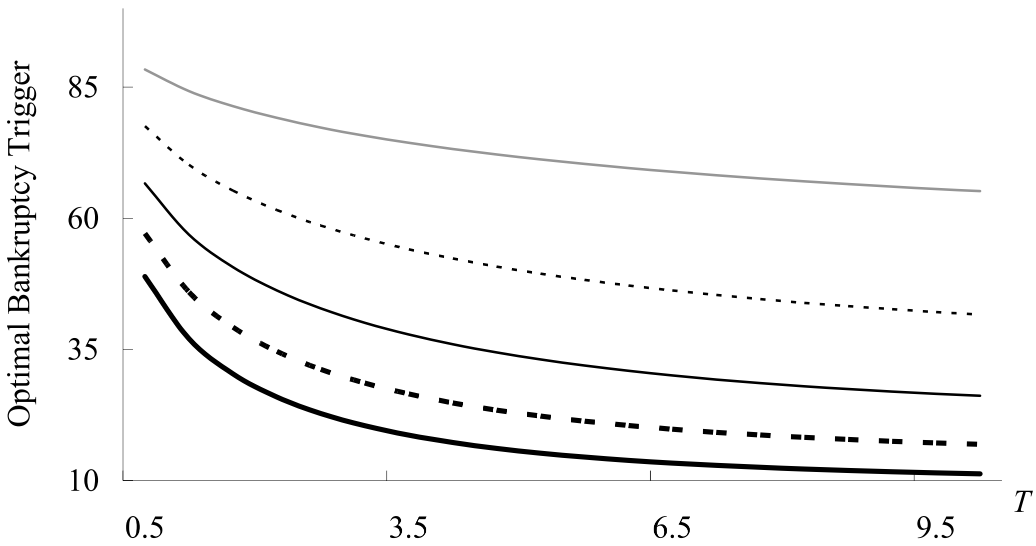

Figure 5

Optimal bankruptcy triggers as a function of the time to maturity *T* for various return volatilities σ

The lines plot the optimal bankruptcy triggers as a function of the time to maturity with return volatilities of 0.1 (gray line), 0.3 (dashed line), 0.5 (solid line), 0.7 (bold dashed line), and 0.9 (bold solid line). The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\beta = 0.05$, and $\gamma = 0.2$.

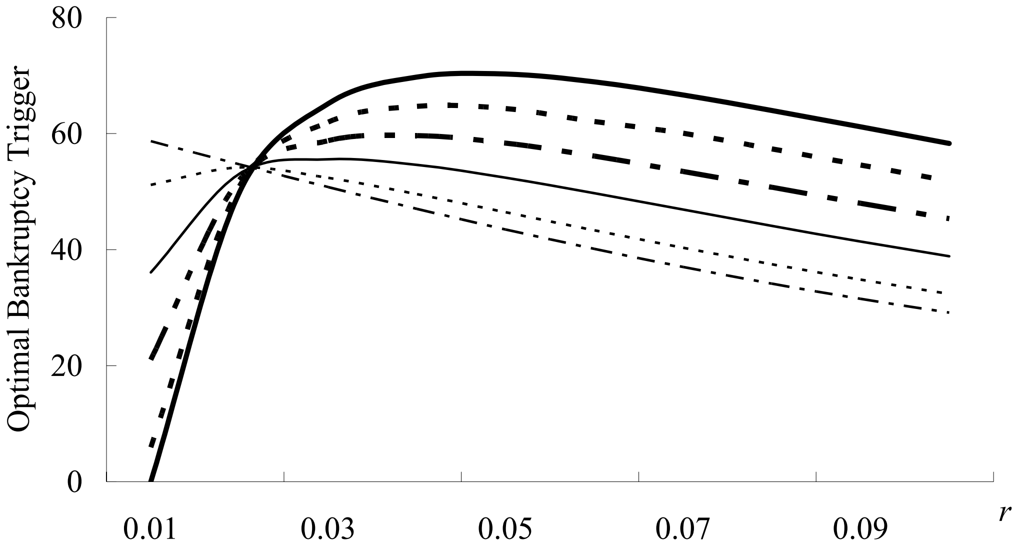

Figure 6

Optimal bankruptcy triggers as a function of the risk-free interest rate r for various coupon payments C

The lines plot the optimal bankruptcy triggers as a function of the risk-free interest rate with coupon payments of 0 (solid-dashed line), 1 (dashed line), 3 (solid line), 5 (bold solid-dashed line), 7 (bold dashed line), and 9 (bold solid line). The parameters are given as follows: $P = 100$, $\tau = 0.35$, $\alpha = 0.5$, $q = 0.04$, $\sigma = 0.2$, $\beta = 0.05$, $\gamma = 0.2$, and $T = 5$.

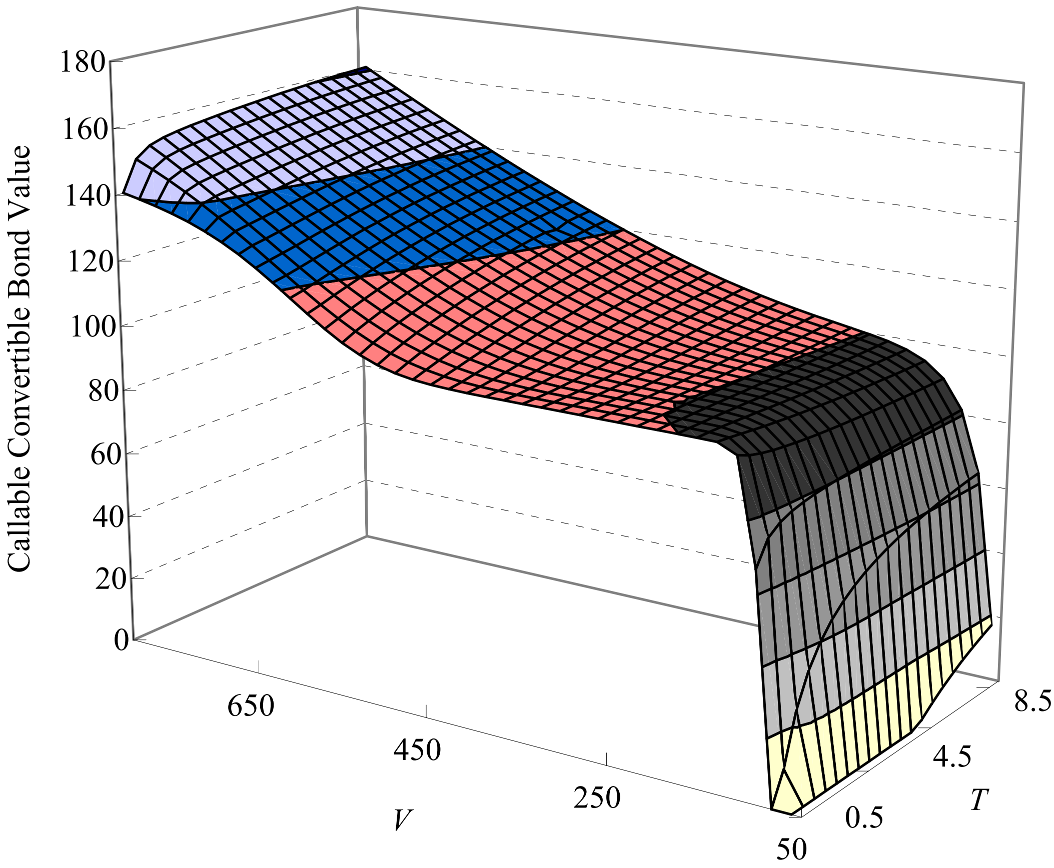

Figure 7

Values of the callable convertible bond as a joint function of the time to maturity T and the unlevered asset value V

The surface plots the value of the callable convertible bonds for varying levels of the unlevered asset values and the time to maturities. The optimal strategies for call, voluntary conversion, and bankruptcy are determined endogenously. The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\sigma = 0.2$, $\beta = 0.05$, and $\gamma = 0.2$.

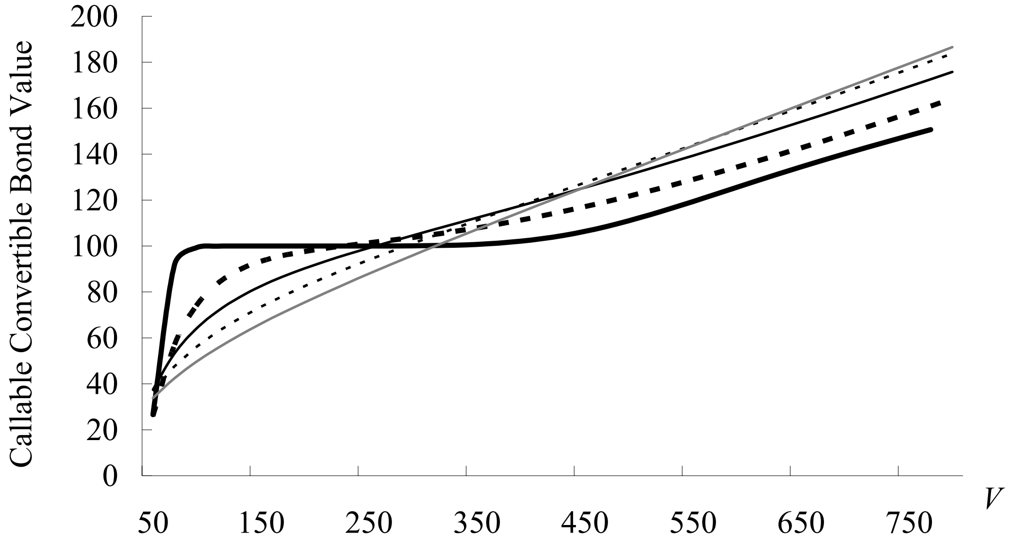

Figure 8

Values of the callable convertible bond as a function of the unlevered asset value V for various return volatilities σ

The lines plot the prices of the callable convertible bond as a function of the unlevered asset value with return volatilities of 0.1 (bold solid line), 0.3 (bold dashed line), 0.5 (solid line), 0.7(dashed line), and 0.9 (gray line). The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\beta = 0.05$, $\gamma = 0.2$, and $T = 5$.

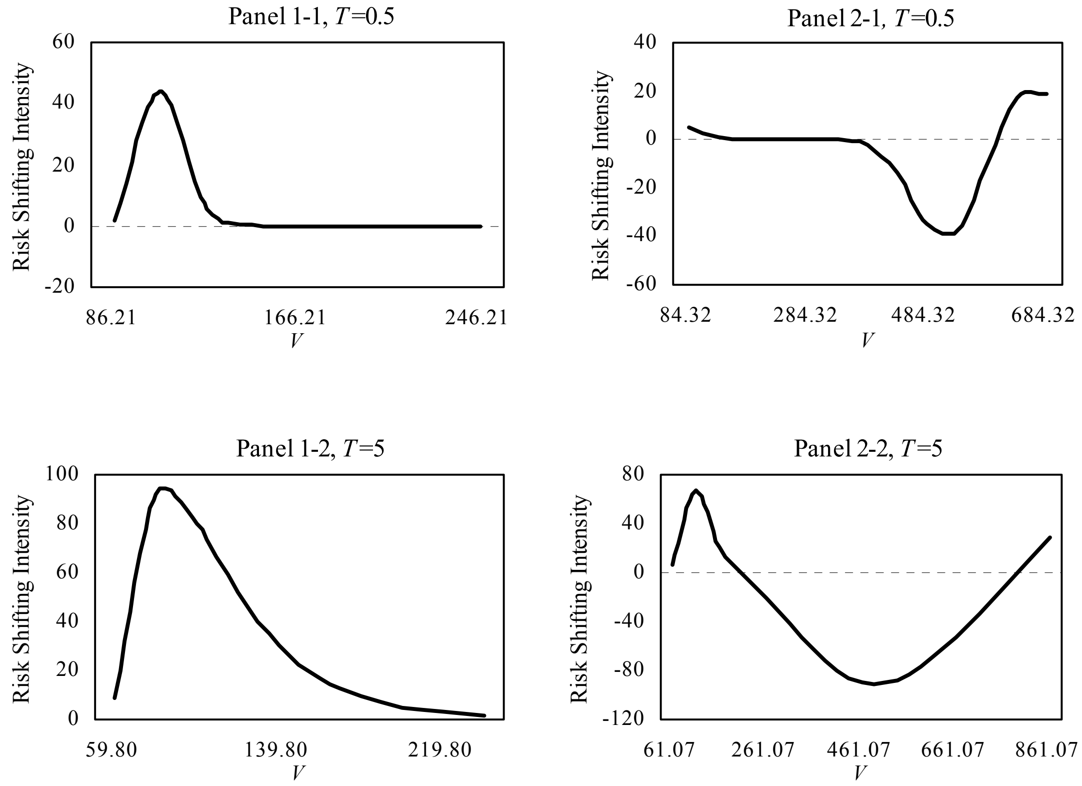

Figure 9

Risk shifting intensities as a function of the unlevered asset value V for the coupon-bond-based model and the callable-convertible-bond-based model

The panels plot risk shifting intensities, which stand for the partial derivatives of the equity value with respect to the return volatility, as a function of the unlevered asset value. Panels 1-1 and 1-2 show risk shifting intensities for the coupon-bond-based model (where the coupon bond is the only debt obligation) with the time to maturities 0.5 and 5, respectively. Panels 2-1 and 2-2 show risk shifting intensities for the callable-convertible-bond-based model (where the callable convertible bond is the only debt obligation) with the time to maturities 0.5 and 5, respectively. The optimal strategies for call, voluntary conversion, and bankruptcy are determined endogenously. The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\sigma = 0.2$, $\beta = 0.05$, $\gamma = 0.2$, and $T = 5$.