Determining Institutional Investor’s Dynamic Asset Allocation

機構投資人的動態資產配置

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摘 要

機構投資人在現今全球的金融市場中佔有舉足輕重的地位，但是在財務理論的領域裏，他們卻是被極度忽略的一群。本文推導出機構投資人的最適動態資產配置模型乃是由標竿避險元素與規模避險元素所組成，其中標竿避險元素表達了機構投資人對標竿投資組合變動的關心程度，而規模避險元素則表達了機構投資人的投資決策受自身規模大小影響的程度。

關鍵詞：動態資產配置、機構投資人。

Abstract

Institutional investors do matter in financial market, but they have been seriously ignored in financial theory. In this paper, we derive a closed-form solution to optimal dynamic asset allocation of institutional investors. We find that the optimal dynamic asset allocation of institutional investors contains two components: the benchmark hedge component and the size hedge component. The benchmark hedge component indicates that institutional investors take care of the volatility of benchmark portfolio. The size hedge component displays the reputation concern of institutional investors.

Keywords: Dynamic asset allocation; Institutional investor.

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I. Introduction

Institutional investors do matter in financial market, but most of the studies on institutional investors have not determined the holdings of different assets by institutional investors. Institutional investors who receive payments and deposits from their customers are also subject to withdrawals from them. With respect to individual investors, institutional investors do indeed bear the extra risk from individual investors.

Institutional investors play no role in the traditional Arrow-Debreu model of resource allocation where individual investors and firms interact through markets. The theory of continuous time finance of (Merton, 1969) provides a link from Arrow-Debreu world to the real world. (Merton, 1969, 1971) studied the issue by first understanding the behavior of an individual who acts as a market price-taker who seeks to maximize the expected utility of end-of-period wealth and the consumption of goods. (Merton, 1969) was the pioneer in using continuous-time modeling in financial economics by formulating the intertemporal consumption and portfolio choice problem of an individual investor in a stochastic dynamic programming setting. Many researchers have extended Merton's dynamic asset allocation model over the past decades.¹ However, all of those extended models still examined the issue by first understanding the behavior of individual investors rather than the behavior of institutional investors. Furthermore, most of the studies on institutional investors, though there are some exceptions (e.g. (Blake, Lehmann, and Timmermann, 1999)), have also not determined the holdings of different assets by institutional investors.

¹ See (Bodie et al., 1992), (Kim and Omberg, 1996), (Brennan et al., 1997), (Sorensen, 1999), (Lioui and Poncet, 2001), (Viceira, 2001), (Xia, 2001), (Brennan and Xia, 2002), and (Bajeux-Besnainou et al., 2003).
Do institutional investors matter? The answer is obviously yes. Surprisingly, in the excellent surveys of asset pricing (Campbell, 2000), continuous time finance (Sundaresan, 2000), and corporate finance (Zingales, 2000), institutional investors were only mentioned in passing. Obviously, institutional investors play only a small role in financial theory. Over the past decades, the equity ownership of investment companies, bank trust departments, insurance companies, foundations, mutual funds and pension funds has grown dramatically. Institutional investors have become increasingly important as equity holders in financial markets. Most people have dealings with some kind of financial institution. Table 1 show that individual investors, in the United States, directly held over 90 percent of corporate equities in 1950, and 68 percent in 1970. The proportion of directly held equities was down to 42 percent in 2000. It has now dropped to even less than 40 percent. The figure for individual ownership in other countries is much lower than that in the United States. At the end of December 1999, it was 19.2 percent in Japan, 21 percent in the United Kingdom, 18.5 percent in Germany, and 24 percent in France.\(^2\) In all of these countries, individual investors play a very limited direct role in equity markets. It is obvious that institutional investors do indeed matter. With the dramatic increase in institutional ownership of equities over the past decades, institutional investors have received increasing attention in academic circles. But most of the studies on institutional investors have also not determined holdings of different assets by institutional investors.

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<td>6.3</td>
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<tr>
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<td>42.0</td>
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<tr>
<td>Other</td>
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<td>142.7</td>
<td>841.4</td>
<td>3,542.6</td>
<td>17,606.5</td>
<td>15,267.1</td>
<td>11,833.9</td>
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Source: Federal Reserve Board "Flow of Funds,"
www.library.louisville.edu/government/federal/agencies/federalreserve/flow.html

\(^2\) See (Bank of Japan, 2000), p. 17.
Standard financial theory assumed that individual investors directly invest their wealth in markets and a person's welfare depends on her end-of-period wealth and the consumption of goods. This theory has become increasingly less appropriate as time has progressed. In the United States, individual investors indirectly investing their wealth in financial markets through institutional investors now number over 60 percent and in other countries the figure for individual ownership is much lower than that of the United States. With respect to the individual investor, the peculiarities of institutional investors are the absence of the opportunity to intertemporally reallocate its consumption and the problem of bequest. Institutional investors who receive payments and deposits from their customers are also subject to withdrawals from their customers. Since both deposits and withdrawals are not under the control of institutional investors, institutional investors bear extra risk from their customers. Moreover, institutional investors do not own the wealth they manage. Hence, the classical consumption-investment problem is inappropriate for institutional investors. Dynamic asset allocation has become an important part of research in financial economics over the last three decades following the pioneering work of (Merton, 1969). Prior to this, relatively little was known about the asset allocation of institutional investors. Based on classical exponential utility function, we derive a closed-form solution of institutional investors' dynamic asset allocation. Although this issue is very important, it has rarely been documented in the literature.

Institutional investors who receive payments and deposits from individual investors are also subject to withdrawals from individual investors. With respect to individual investors, institutional investors do indeed bear the extra risk from individual investors. The main contributions of this paper are taking the extra risk into account and deriving a closed-form solution to the dynamic asset allocation of institutional investors. We determine the extra risk via empirical results and find that the optimal dynamic asset allocation of institutional investors contains two components: the benchmark hedge component and the size hedge component. There are three major differences between our model and others. First, the benchmark hedge component has never been shown in other models. The benchmark hedge component shows that the institutional investor
takes care of the volatility of the benchmark portfolio and deduces that the greater the volatility of the benchmark portfolio, the greater the amount of risky assets held by institutional investors. Second, the size hedge component deduces that the optimal dynamic asset allocation of institutional investors is inversely proportional with its total net managed asset. This displays the phenomenon of reputation concern that has previously been argued in the literature. Finally, both the benchmark hedge component and the size hedge component are influenced by the extra risk from individual investors. The factor of extra risk from individual investors has never been documented in the literature.

The remainder of this paper is organized as follows. Section 2 depicts the economy, section 3 derives our model and section 4 describes the economic implications and concludes.

II. The economy

1. Relationship between return and flow

Institutional investors who receive payments and deposits from their customers are also subject to withdrawals from their managed assets. More importantly, both deposits and withdrawals are not under the control of institutional investors. They are subject to a particular kind of risk represented by the withdrawals from and contributions to the managed assets. The particular kind of risk that institutional investors have faced can be described by its performance. Individual investors of funds delegate its productive decisions to institutional investors, who cannot usually observe these decisions directly but can infer them from the fund performance and invest accordingly. It is well documented that individual investors of managed funds chase positive performance. Individuals tend to move cash into the managed funds that had the
highest return in the preceding year. It is reasonably argued that institutional investors should also be sensitive to the change in wealth of the fund they managed since institutional investors pick stocks and individual investors pick institutions. In short, the particular kind of risk represented by the withdrawals from and contributions to the managed assets that institutional investors have faced is influenced by its performance.

The information content of fund returns is one of the most popular topics in finance. Previous evidence suggests a positive relation between returns and cash flows of managed funds. Many authors have found that performance is an important determinant of cash flows in managed funds and several papers have presented evidence that a flow-performance relationship does indeed exist. 3 Academic literature documents a convex relation between returns and fund flows of open-end mutual funds. (Ippolito, 1992), (Sirri and Tufano, 1998), and (Lynch and Musto, 2003) all find a small positive slope in the lower region and a considerably larger slope in the higher region, which means that net new investment to be much less sensitive to returns in the region of bad returns. The open-end mutual fund flow-performance relation is highly convex, implying that open-end mutual fund investors disproportionately flock to good performers but do not punish poor performers by withdrawing assets symmetrically. 4 (Guercio and Tkac, 2002) found that the empirical distribution of open-end mutual fund flows appears to be asymmetric. The top 5% experience net inflows nearly three times larger than the outflows in the bottom 5%. (Chevalier and Ellison, 1997) demonstrate that the shape of the flow-performance relationship for young mutual funds is quite steep and close to being linear. A 1% rise in return of an

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3 (Ippolito, 1992), (Patel, Zeckhauser, and Hendricks, 1994), (Gruber, 1996), (Chevalier and Ellison, 1997), (Sirri and Tufano, 1998), (Zheng, 1999), and (Guercio and Tkac, 2002) all find that past performance is an important determinant of flow in an open-end mutual fund.

4 According to the disposition effect (one implication extended from (Kahneman and Tversky, 1979)'s prospect theory and labeled by (Shefrin and Statman, 1985)), individual investors have the tendency to realize their winners too soon and hold losers too long. For the overconfidence effect ((Odean, 1998) and (Gervais and Odean, 2001)), high returns make investors overconfident and overconfident investors increase their volume of trading. When fund past returns are negative, the disposition effect argues that the existing investors have the tendency to hold losers. In the meantime, funds with bad performance are less attractive for potential investors. In short, existing investors are unwilling to sell and potential investors are unwilling to buy when fund performance is bad. On the contrary, if fund past returns are superior, the disposition effect argues that the existing investors have the tendency to realize their winners. In the meantime, funds with superior performance are much attractive for potential investors. Although the disposition effect argues existing investors have the tendency to realize their winners, there are lots of potential investors who follow the overconfidence effect and are willing to own the shares. The strong positive relationship between current volume and lagged return provides empirical support for the overconfidence hypothesis.
average young fund is associated with about a 4% increase in the fund's annual relative flow. In contrast, old funds have a generally convex shape.

In contrast to mutual fund clients who invest only on their own behalf, (Lakonishok, Shleifer and Vishny, 1992) argue that pension sponsor officials acting as fiduciaries have agency problems that induce them to value manager characteristics that are easily justifiable to superiors or a trustee committee. Pension fund sponsors are often finance professionals and most pension sponsors rely heavily on the recommendations of consultants. This implies pension fund sponsors are more financially sophisticated than mutual fund investors. (Guercio and Tkac, 2002) also document that the empirical distribution of pension fund flows is more symmetric than open-end mutual fund flows. The bottom 5% of pension funds actually suffer larger dollar outflows than the top 5% gains, $524 million in outflows versus $400 million in inflows. In contrast with the open-end mutual funds segment, the flow-performance relation is approximately linear for the pension funds segment.

It is generally believed that the performance of the benchmark is important because its aids in measuring the investment performance of institutional investors and provide both clients and trustees with a reference point for monitoring that particular performance. The performance of the benchmark has important implications for how the actions of the institutional investors are monitored and interpreted. Beginning with (Jensen, 1968, 1969), the evaluation of managed fund performance has generated a great deal of interest in academic circles and a variety of evaluation techniques have been proposed and implemented. (Roll, 1978) has argued that performance evaluation with the benchmark portfolio is likely to be sensitive to the benchmark choice. (Grinblatt and Titman, 1994) have also argued that the performance of institutional investors is sensitive to the choice of benchmark portfolios. They found that the choice of a benchmark could have a large effect on inferences concerning performance.

**Assumption 1:** The relative flow is positively linearly related to performance.

Institutional investors who receive payments and deposits from their
customers do indeed face a particular kind of risk represented by the withdrawals from and contributions to the managed assets. The sensitivity of relative flow to performance is the measurement of the particular kind of risk. For the sake of simplicity, it is assumed that the relationship between relative flow and performance is linear and has the following characteristic:

\[ \text{flow}_t = \frac{\text{FLOW}_t}{\text{TNA}_t} = \text{SFP} \times (R_t - R_{BM_t}) \]  

where

- \( \text{flow}_t \) is the relative flow at time \( t \).
- \( \text{FLOW}_t \) is the total net cash flow at time \( t \).
- \( \text{TNA}_t \) is the total net managed assets of the institutional investor at time \( t \).
- \( \text{SFP} \) is the sensitivity of relative flow to performance, \( \text{SFP} > 0 \).
- \( R_t \) is the fund's return at time \( t \).
- \( R_{BM_t} \) is the rate of return of relative benchmark portfolio at time \( t \).

2. Investment opportunity sets

It is assumed that there are \( N \) risky securities and one risk-free security in the market. All of these securities can be divided infinitely and the returns of each asset accrue only in the form of capital gains (no dividend payout). There are no taxes, no transaction costs and no short-sell constraints in the market. Moreover, the institutional investor is also permitted to short-sell.

It is assumed that the price of the \( j \)th security at time \( t \), \( S_{jt} \), follows Ito process that has the following differential equation.

\[ \frac{dS_{jt}}{S_{jt}} = \mu_j dt + \sigma_j dz_j \]  

where

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5 It is much more sophisticated in our model when the relationship between past returns and fund flows is convex. Besides, (Chevalier and Ellison, 1997) demonstrate that the shape of the flow-performance relationship for young mutual funds is quite steep and close to linear. (Guercio and Tkac, 2002) had also documented that the flow-performance relationship is approximately linear in the pension fund segment. This issue remains a challenge for future researchers.
\( z_j \) is a Wiener process.
\( \mu_j \) is the expected rate of return of the \( j \)th risky security at time \( t \).
\( \sigma_j \) is the stock index volatility.

Let \( BM_t \) be the net value of benchmark portfolio at time \( t \) and the dynamics for \( BM_t \) is given by the following equation.

\[
\frac{d(BM_t)}{BM_t} = \mu_{BM} dt + \sigma_{BM} dz_{BM}
\]  

where

\( z_{BM} \) is a Wiener process.
\( \mu_{BM} \) is the expected rate of return of the relative benchmark portfolio at time \( t \).
\( \sigma_{BM} \) is the benchmark portfolio index volatility.

Let \( B_t \) be the total amount of the risk-free security that the institutional investor holds at time \( t \) and the dynamics for \( B_t \) are given by

\[
\frac{dB_t}{B_t} = r dt
\]  

where \( r \) is the expected rate of return of the risk-free security.

3. Management fee

Benchmarks are important, and so are fee structures. Prior to 1971, there were three different forms of fee schedules in the United States:

(1) Percentage management fee schedule: a percentage of fund assets.

(2) Symmetric performance-based fee schedule: a percentage of fund assets plus (minus) a preset percentage of the gain (loss) in fund value measured relative to a benchmark portfolio.
(3) Asymmetric performance-based fee schedule: the same as (2) except that the percentage share of the gain exceeded the percentage share of the loss.

Fearing that fund managers compensated by asymmetric fees would take on too much risk, the U.S. Congress and the Securities and Exchange Commission (SEC) prohibited the use of asymmetric performance-based fee schedules in 1971. More recently, the SEC has reintroduced asymmetric performance fees. For simplicity, it is assumed that the institutional investor adopts the percentage management fee schedule.

**Assumption 2:** *The management fee is positively linearly related to total net managed assets.*

At time $t$, the management fee ($MF_t$) is defined by the following equation.

$$MF_t = \theta \times TNA_t$$

(5)

where $\theta$ is determined by the institutional investor, $0 < \theta < 1$.

### III The model

1. Utility function of the institutional investor

   The utility function of the institutional investor is defined as follows.

   $$U(MF_t) = -e^{-\beta \times MF_t}$$

(6)

where $\beta$ is the constant absolute risk aversion coefficient, $0 < \beta < 1$.

This is a well-known negative exponential utility function, i.e., $U'(MF_t) > 0$ and $U''(MF_t) < 0$. 
2. Derivations of the model

Since the institutional investor has to allocate the total net managed assets \( (TNA_t) \) into the market, it is assumed that the institutional investor constructs a portfolio as

\[
TNA_t = \sum_{j=1}^{N} n_j \beta_j + B_t
\]

where \( n_j \) is the number of shares of the \( j \)th risky security held by the institutional investor at time \( t \).

The dynamic stochastic process of total net managed assets at time \( t \) is defined as

\[
d(TNA_t) = (1 + SFP) \left( \sum_{j=1}^{N} w_j \left( (\mu_j - r) dt + \sigma_j dz_j \right) \times TNA_t + r \times TNA_t \times dt \right) \\
- MF_t \times dt - SFP \times (\mu_{BM} dt + \sigma_{BM} dz_{BM}) \times TNA_t
\]

It is assumed that the institutional investor aims at solving the dynamic portfolio choice problem of

\[
\text{Max}_{w_t} \ E_t \left[ \int_t^\infty e^{-\delta \tau} U(MF_t) \ d\tau \right]
\]

s.t. equations (5), (6), (7), and (8).

where

- \( w_t \) is the \( 1 \times N \) vector with representative elements \( w_{jt} \).
- \( w_{jt} \) is the proportion of total net managed assets that the institutional investor invests in the \( j \)th risky security at time \( t, j = 1, \ldots, N \).
- \( E_t \) is the expectation operator at time \( t \).
- \( \delta \) is the rate of time preference of the institutional investor.

Let \( J = J(TNA_t, t) \) be a well-behaved function such that
The Bellman’s equation is

\[ J = \max_{w_t} E_t \left[ \int_t^\infty e^{-\delta \tau} U(MF_{\tau}) \, d\tau \right] \tag{10} \]

The Bellman’s equation is

\[
0 = \max_{w_t} \left[ e^{-\delta t} U(MF_t) + J_t \right. \\
+ J_t \left[ (1 + SFP) TNA_t w_t \mu + r \times TNA_t - MF_t - SFP \times \mu_{BM} \times TNA_t \right] \\
+ \frac{1}{2} J_t \left[ (w_t \Sigma w_t)' (1 + SFP)^2 TNA_t^2 \right. \\
- J_{TT} \times SFP \times (1 + SFP) \left. TNA_t^2 \right] \tag{11} \]

where

- \( w_t' \) is the transposition of \( w_t \).
- \( \mu \) is the \( N \times 1 \) vector of expected rate of excess return of risky assets.
- \( \Sigma \) is the \( N \times N \) variance-covariance matrix of stock index volatility.
- \( \sigma \) is the \( N \times 1 \) vector of stock index volatility.
- \( J_T \) and \( J_t \) denote the derivatives of \( J \) with respect to \( TNA_t \) and \( t \), respectively.

We use the similar notation for higher derivatives.

The optimal dynamic asset allocation of the institutional investor can then be characterized by following equation.

\[
w_t' = \frac{SFP \times \sigma_{BM} \Sigma^{-1} \sigma}{1 + SFP} + \frac{1}{\beta(1 + SFP) \times TNA_t} \Sigma^{-1} \mu \tag{12} \]

3. The special case of \( N = 1 \)

If \( N = 1 \), the model will have the following degenerate form.

\[
w_t' = \frac{SFP \sigma_{BM} \sigma}{\sigma^2} + \frac{1}{\beta(1 + SFP) \times TNA_t} \frac{\mu - r}{\sigma^3} \tag{12a} \]
4. Discussion of the results

The main results of this paper can be generalized as follows.

**Result 1:** *Optimal dynamic asset allocation of the institutional investor contains two components: the benchmark hedge component and the size hedge component.*

It is clear that there are two components in equation (12). The first term is labeled the "benchmark hedge component" since it is an affine function of the benchmark portfolio. The position of the benchmark hedge component must always be positive. The second term is labeled the "size hedge component" since it is an affine function of total net managed assets. The position of the risk aversion hedge component can be either positive or negative, depending on the expected rate of excess return.

**Result 2:** *The benchmark hedge component indicates that the institutional investor takes care of the volatility of the benchmark portfolio.*

Equation (12) shows that the greater the expected standard deviation of the benchmark portfolio, the greater the number of risky assets held by the institutional investor.

**Result 3:** *The size hedge component depicts the reputation concern of the institutional investor.*

By equation (12a), the volatility of the portfolio held by institutional investors is inversely proportional with its total net managed assets. This is consistent with the results of empirical evidence.

**Result 4:** *Both the benchmark hedge component and the size hedge component are influenced by the sensitivity of relative flow to performance.*

This is the first study to introduce the sensitivity of relative flow to performance into the dynamic asset allocation model. The model figures that the benchmark hedge component is directly proportional with the sensitivity, and the size hedge component is inversely proportional with the sensitivity. Unfortunately, the exact total effect of the sensitivity is unclear in this model.
IV Economic implications and conclusions

In this paper, we derive a closed-form solution to the dynamic asset allocation of institutional investors. This issue has never been covered in the literature. This model pictures that the optimal dynamic asset allocation of the institutional investors contains two components: the benchmark hedge component and the size hedge component. The benchmark hedge component shows that the greater the benchmark portfolio index volatility, the greater the amount of risky assets held by the institutional investor. In other words, the volatility of the benchmark portfolio is an important factor to consider when holding risky assets. This makes sense since the greater the benchmark portfolio index volatility, the greater the probability that the institutional investor will outperform or underperform the benchmark portfolio. In order to avoid falling behind a peer group, the institutional investor has to take care of the volatility of the benchmark portfolio. The size hedge component not only displays the impact of the risk aversion coefficient, but also figures the influence of total net managed assets. The implication of the influence of total net managed assets may result from reputation concern.

Moreover, there is an additional adjustment factor which is determined by the risk of flow. The information content of fund returns is a popular topic in finance and many authors have found that performance is an important determinant of cash flows in managed funds. This study is the first to determine the extra risk via empirical results and introduce the extra risk into dynamic asset allocation. We find that both the benchmark hedge component and the size hedge component are influenced by the sensitivity of relative flow to performance. Unfortunately, the total effect of the sensitivity is unclear in this model. These phenomena have never been mentioned in the literature and remains as future challenges for researchers in this field.
References


