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A REVISED METHOD FOR THE FORCE OF MORTALITY BY USING THE NUMERICAL DIFFERENTIATION OF LAGRANGE'S INTERPOLATION

HWAI-NIEN YANG

Professor

Department of International Trade

National Chengchi University

INTRODUCTION

The mortality function μ_x, defined by the formula (1, P. 13)

$$\mu_x = \frac{-1}{l_x} Dl_x, \tag{1}$$

is usually referred to as the force of mortality at age x, where l_x represents a function of the number living or surviving at age x. Since it is difficult to find a mathematical expression for the function l_x, usually an empirically decreasing function, numerical differentiation is therefore necessary in determining the numerical value of μ_x.

Approximate numerical values of μ_x would be got by using the following formulas:

1) μ_x is derived from the formula (1, P. 18).

$$\mu_x = \frac{1}{2l_x}(l_{x-1} - l_{x+1}) \tag{2}$$

which will be exact, if l_x is a polynomial of the second degree.

2) μ_x is frequently employed by compilers in the building up of a life table, derived from the formula (1, P.18, 2, P.5)

$$\mu_x=\frac{1}{12l_x}[8(l_{x-1}-l_{x+1})-(l_{x-2}-l_{x+2})] \tag{3}$$

which will be exact, if l_x is a polynomial of the fourth degree.

Both the formulas (2) and (3) can be obtained by expanding l_{x+k}, $k=-2, -1, 1, 2,$ in Taylor series about x, and eliminating the terms of more than the first order derivatives of l_x.

3) μ_x is derived from the formula (1, P.18) by using the relationship of operator E between the derivative operator D, and the finite forward difference operator Δ, where $E = e^D$ and $E = 1 + \Delta$.

Hence, we have

$$D = \ln(1+\Delta)$$

$$= \Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + \cdots \tag{4}$$

and

$$\mu_x = \frac{-1}{l_x}(\Delta l_x - \frac{1}{2}\Delta^2 l_x + \frac{1}{3}\Delta^3 l_x - \frac{1}{4}\Delta^4 l_x + \cdots). \tag{5}$$

The objective of this research is to determine the force of mortality function μ_x, by using the numerical differentiation of Lagrange's interpolation formula at a certain age x, and to demonstrate that the formula (3) can be used with l_x being a polynomial of the fourth degree or less than the fourth.

PRELIMINARIES AND NOTATIONS

The following notations and formulas will be used throughout this research:

N_1 l_x is a continuously decreasing function of the number of lives surviving as x increases, defined with the interval $0 \le x \le w$, where x represents the life expectancy and $l_w = 0$.

N_2 d_x represents the number of deaths, derived from the equation

$$d_x = l_x - l_{x+1}, \text{ for } x = 0, 1, 2, 3, \ldots, w, \tag{6}$$

and thus the formulas (2) and (3) can be changed respectively into the following forms:

$$\mu_x = \frac{1}{21_x}(d_{x-1} - d_x), \tag{7}$$

and $$\mu_x = \frac{1}{121_x}[7(d_{x-1} + d_x) - (d_{x-2} + d_{x+1})]. \tag{8}$$

N_3 D is the derivative operator.

N_4 Δ is the finite forward difference operator, defined by the equation

$$\Delta U_x = U_{x+1} - U_x, \text{ for } x = 0, 1, 2, 3, \ldots \tag{9}$$

N_5 Δ^m, m = 0, 1, 2, 3, . . . , represents the mth order finite forward difference operator, defined by the formula (3, P.157):

$$\Delta^m U_x = \sum_{k=0}^{m} \binom{m}{k} U_{x+k}, \tag{10}$$

where $\Delta^m U_x = n!$, (11)

if U_x is a polynomial of the nth degree and $n = m$(3, P.158),

and $\Delta^m U_x = 0$, (12)

if $n < m$.

N_6 E is the shifting operator defined by the formula (2, P. 8)

$$\begin{aligned} EU_x &= U_{x+1} \\ &= U_x + \Delta U_x \\ &= (1 + \Delta) U_x, \end{aligned} \tag{13}$$

and $$\begin{aligned} EU_x &= U_x + DU_x + \frac{1}{2!} D^2U_x + \frac{1}{3!}D^3U_x + \cdots \\ &= (1 + D + \frac{1}{2!}D^2 + \frac{1}{3!}D^3 + \cdots) U_x \\ &= e^D U_x. \end{aligned} \tag{14}$$

The formulas (4) and (5) can be obviously obtained by the operators D, Δ, and E.

N_7 Lagrange's Interpolation formula.

Assume that a function U_x is continuously differentiable n+1 times and n+1 points (x_k, U_{x_k}), for $x_k = x_0 + k$, and k = 0, 1, 2, 3, . . . , n are known, then the function U_x can be approximated with Lagrange's interpolation formula (3, P.165) and we shall have

$$U_x=\sum_{k=0}^{n} P_k(x)U_{x_k}, \tag{15}$$

where $P_k(x) = \dfrac{(x - x_0)(x - x_1)\cdots(x - x_{k-1})(x - x_{k+1})\cdots(x - x_n)}{(x_k - x_0)(x_k - x_1)\cdots(x_k - x_{k-1})(x_k - x_{k+1})\cdots(x_k - x_n)}$.

N8 $U'(x_r)$ (3, P. 166) represents the derivative of $\sum_{k=0}^{n} P_k(x)U_{x_k}$ at $x = x_r$, for $r = 0, 1, 2, 3, \ldots, n$, defined by the equation:

$$U'(x_r) = \sum_{r \neq k}^{n} \frac{1}{x_r - x_k} [U_{x_r} - \frac{F'(x_r)}{F'(x_k)} U_{x_k}], \tag{16}$$

where $F(x) = (x - x_0)(x - x_1)\cdots(x - x_k)\cdots(x - x_n)$,

$F_k(x) = (x - x_0)(x - x_1)\cdots(x - x_{k-1})(x - x_{k+1})\cdots(x - x_n)$;

$F'(x) = (x - x_k)F'_k(x) + F_k(x)$.

RESULTS

1) Assume that the function l_x is continuously differentable 3 times and 3 points (x_0, l_{x_0}), (x_1, l_{x_1}) and (x_2, l_{x_2}) are given, then l_x can be approximated with Lagrange's interpolation formula (15) and we shall have

$$l_x=\frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}l_{x_0} + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}l_{x_1} + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}l_{x_2},$$

where $x_1 = x_0 + 1$ and $x_2 = x_0 + 2$. (17)

From (16), the following derivative of (17) at x_0 and x_1 can be obtained respectively

$$l'_{x_0} = -\frac{3}{2}l_{x_0} + 2l_{x_1} - \frac{1}{2}l_{x_2}, \tag{18}$$

and $$l'_{x_1} = -\frac{1}{2}l_{x_0} + \frac{1}{2}l_{x_2}. \tag{19}$$

Hence, from (1), the force of mortality μ_x can be obtained as follows:

$$\mu_{x_0} = \frac{1}{2l_{x_0}} (3l_{x_0} - 4l_{x_1} + l_{x_2}), \tag{20}$$

and $$\mu_{x_1} = \frac{1}{2l_{x_1}}(l_{x_0} - l_{x_2}). \tag{21}$$

In general, replacing x_0, x_1 and x_2 with x, x+1 and x+2 in (20), we have

$$\mu_x = \frac{1}{2l_x}(3l_x - 4l_{x+1} + l_{x+2}), \tag{22}$$

or from (6),

$$\mu_x = \frac{1}{2l_x}(3d_x - d_{x+1}). \tag{23}$$

Replacing x_0, x_1 and x_2 with x-1, x and x+1, in (21), we have

$$\mu_x = \frac{1}{2l_x}(l_{x-1} - l_{x+1}) \tag{24}$$

which is exactly the formula (2),
or from (6)

$$\mu_x = \frac{1}{2l_x}(d_{x-1} + d_x). \tag{25}$$

2) Assume that the function l_x is continuously differentable 4 times and 4 points (x_0, l_{x_0}), $(x_1\ l_{x_1})$, (x_2, l_{x_2}) and (x_3, l_{x_3}) are given, then l_x can be approximated by using the formula (15) and we have

$$l_x = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x1)(x_0-x_2)(x_0-x_3)}l_{x_0} + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_0)(x_1-x_3)}l_{x_1}$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}l_{x_2} + \frac{(x-x0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}l_{x_3} \tag{26}$$

where $x_1 = x_0+1$, x_2+2 and $x_3 = x_0+3$.

From (16), the following derivatives of (26) at x_0 and x_1 can be obtained respectively

$$l'_{x_0} = -\frac{11}{3!}l_{x_0} + 3l_{x_1} - \frac{3}{2!}l_{x_2} + \frac{2}{3!}l_{x_3}, \tag{27}$$

and
$$l'_{x_1} = -\frac{2}{3!}l_{x_0} - \frac{1}{2}l_{x_1} + l_{x_2} - \frac{1}{3!}l_{x_3} \tag{28}$$

Hence from (1), we have

$$\mu_{x_0} = \frac{1}{6l_{x_0}}(11l_{x_0} - 18l_{x_1} + 9l_{x_2} - 2l_{x_3}), \tag{29}$$

and $$\mu_{x_1} = \frac{1}{6l_{x_1}}(2l_{x_0} + 3l_{x_1} - 6l_{x_2} + l_{x_3}). \tag{30}$$

In general, replacing x_0, x_1, x_2, and x_3 with x, $x+1$, $x+2$ and $x+3$ in (29), we have

$$\mu_x = \frac{1}{6l_x}(11l_x - 18l_{x+2} - 9l_{x+2} - 2l_{x+3}), \tag{31}$$

or from (6),

$$\mu_x = \frac{1}{6l_x}(11d_x - 7d_{x+1} + 2d_{x+2}). \tag{32}$$

Replacing x_0, x_1, x_2 and x_3 with $x-1$, x, $x+1$ and $x+2$ in (30), we have

$$\mu_x = \frac{1}{6l_x}(2l_{x-1} + 3l_x - 6l_{x+1} + l_{x+2}), \tag{33}$$

or from (6),

$$\mu_x = \frac{1}{6l_x}(2d_{x-1} + 5d_x - d_{x+1}). \tag{34}$$

3) Assume that the function l_x is continuously differentiable 5 Times, and 5 points (x_0, l_{x_0}), (x_1, l_{x_1}), (x_2, l_{x_2}), (x_3, l_{x_3}) and (x_4, l_{x_4}) are given, than l_x can be approximated by using the formula (15) and we have

$$\begin{aligned} l_x = & \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} l_{x_0} \\ & + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} l_{x_1} \\ & + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} l_{x_2} \\ & + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} l_{x_3} \\ & + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} l_{x_4}, \end{aligned} \tag{35}$$

where $x_1 = x_0+1$, $x_2 = x_0+2$, $x_3 = x_0+3$ and $x_4 = x_0+4$.

From (16), the following derivatives of (35) at x_0, x_1 and x_2 can be obtained respectively:

$$l'_{x_0} = -\frac{50}{4!} l_{x_0} + \frac{24}{3!} l_{x_1} - \frac{12}{2!2!} l_{x_2} + \frac{8}{3!} l_{x_3} - \frac{6}{4!} l_{x_4}, \quad (36)$$

$$l'_{x_1} = -\frac{6}{4!} l_{x_0} - \frac{5}{3!} l_{x_1} - \frac{6}{2!2!} l_{x_2} - \frac{3}{3!} l_{x_3} - \frac{2}{4!} l_{x_4}, \quad (37)$$

$$l'_{x_2} = \frac{2}{4!} l_{x_0} - \frac{4}{3!} l_{x_1} + \frac{4}{3!} l_{x_3} - \frac{2}{4!} l_{x_4}. \quad (38)$$

Hence, from (1), we have

$$\mu_{x_0} = \frac{1}{12 l_{x_0}} (25 l_{x_0} - 48 l_{x_1} + 36 l_{x_2} - 36 l_{x_3} - 3 l_{x_4}), \quad (39)$$

$$\mu_{x_1} = \frac{1}{12 l_{x_1}} (3 l_{x_0} + 10 l_{x_1} - 18 l_{x_1} + 6 l_{x_3} - l_{x_4}), \quad (40)$$

and

$$\mu_{x_2} = \frac{1}{12 l_{x_2}} (-l_{x_0} + 8 l_{x_1} \qquad - 8 l_{x_3} + l_{x_4}). \quad (41)$$

In generrl, replacing x_0, x_1, x_2, x_3 and x_4 with x, x+1, x+2, x+3 and x+4 in (39), we have

$$\mu_x = \frac{1}{12 l_x} (25 l_x - 48 l_{x+1} + 36 l_{x+2} - 16 l_{x+3} - 3 l_{x+4}), \quad (42)$$

or from (6),

$$\mu_x = \frac{1}{12 l_x} (25 d_x - 23 d_{x+1} + 13 d_{x+2} - d_{x+3}). \quad (43)$$

Replacing x_0, x_1, x_2, x_3 and x_4 with x - 1, x, x+1, x+2 and x+3 in (40), we have

$$\mu_x = \frac{1}{12 l_x} (3 l_{x-1} + 10 l_x - 18 l_{x+1} + 6 l_{x+2} - l_{x+3}), \quad (44)$$

or from (6),

$$\mu_x = \frac{1}{12 l_x} (3 d_{x-1} + 13 d_x - 5 d_{x+1} + d_{x+2}). \quad (45)$$

Replacing x_0, x_1, x_2, x_3 and x_4 with x-2, x-1, x, x+1 and x+2 in (41), we have

$$\mu_x = \frac{1}{12l_x}(-l_{x-2} + 8l_{x-1} - 8l_{x+1} + l_{x+2}) \tag{46}$$

which is exacily the formula (3), or from (6)

$$\mu_x = \frac{1}{12l_x}[7(d_{x-1} + d_x) - (d_{x-2} + d_{x+1})]. \tag{47}$$

Obviously, the formulas (22), (23), (24), (25), (31), (32), (33), (34), (42), (43), (44), (45), (46) and (47) can be used to find the approximate value of the force of mortality μ_x under the assumptions of each.

REMARKS

1) By using the formula (10), the formula (31) cau be changed into another form as follows:

$$\begin{aligned}\mu_x &= \frac{1}{6l_x}(11l_x - 18l_{x+1} + 9l_{x+2} - 2l_{x+3}) \\ &= \frac{1}{6l_x}(2l_x - 6l_{x+1} + 6l_{x+2} - 2l_{x+3} + 9l_x - 12l_{x+1} - 3l_{x+2}) \\ &= \frac{1}{3l_x}\Delta^3 l_x + \frac{1}{2l_x}(3l_x - 4l_{x+1} + l_{x+2}),\end{aligned} \tag{48}$$

or from (6),

$$\mu_x = \frac{1}{3l_x}\Delta^3 l_x + \frac{1}{2l_x}(3d_x - d_{x+1}). \tag{49}$$

If l_x is a polynomial of the second degree, the formula (31) can be also used to find the value of μ_x, because as it is obvious from (12), the $\Delta^3 l_x$ is equal to zero in (48) and (49).

If l_x is a polynomial of the third defree, the formula (48) and (49) can be written in the following forms respectively:

$$\mu_x = \frac{1}{2l_x}(3l_{x+1} - 4l_{x+1} + l_{x+2}) \tag{50}$$

and
$$\mu_x = \frac{1}{2l_x}(3d_x - d_{x+1}), \tag{51}$$

Here to find the approximate value of μ_x, the term $\frac{1}{3l_x}\Delta^3 l_x$ can be deleted, because a very small constant could be changed into $\frac{2}{l_x}$, when l_x is veay large. Hence, it is obvious that the formulas (50) and (51) are exactly the formulas (22) and (23) respectively.

2) Similarly, the formulas (42) and (46) can be changed into the following forms respectively:

$$\mu_x = \frac{1}{12l_x}(25l_x - 48l_{x+1} + 36l_{x+2} - 16l_{x+3} + 3l_{x+4})$$

$$= \frac{1}{12l_x}(3l_x - 12l_{x+1} + 18l_{x+2} - 12l_{x+3} + 3l_{x+4} + 22l_x - 36l_{x+1} + 18l_{x+2} - 4l_{x+3}) \quad (52)$$

or form (6),

$$\mu_x = \frac{1}{4l_x}\Delta^4 l_x + \frac{1}{6l_x}(11d_x - 7d_{x+1} + 2d_{x+2}), \quad (53)$$

and

$$\mu_x = \frac{1}{12l_x}(-l_{x-2} + 8l_{x-1} - 8l_{x+1} + l_{x+2})$$

$$= \frac{1}{12l_x}(-l_{x-2} + 4l_{x-1} - 6l_x + 4l_{x+1} - l_{x+2} + 4l_{x-1} + 6l_x - 12l_{x+1} + 2l_{x+2})$$

$$= \frac{-1}{12l_x}\Delta^4 l_{x-2} + \frac{1}{6l_x}(2l_{x-1} + 3l_x - 6l_{x+1} + l_{x+2}), \quad (54)$$

or from (6),

$$\mu_x = \frac{-1}{12l_x}\Delta^4 l_{x-2} + \frac{1}{6l_x}(d_{x-1} + 3d_x - d_{x+1}). \quad (55)$$

If l_x is a polynomial of the second degree for case and third degree for another, the formulas (45) and (47) can be also used to find the value of μ_x; because as it is obvious from (12) $\Delta^4 l_x$ and $\Delta^4 l_{x-2}$ are equal to zero in (52), (53), and, (54) and (55).

If l_x is a polynomial of the fourth degree, the formulas (52), (53), (54) and (55) can be written in the following forms respectively:

$$\mu_x = \frac{1}{6l_x}(11l_x - 18l_{x+1} + 9l_{x+2} - 2l_{x+3} - 2l_{x+3}), \quad (56)$$

$$\mu_x = \frac{1}{6l_x}(11d_x - 7d_{x+1} + 2d_{x+2}), \quad (57)$$

$$\mu_x = \frac{1}{6l_x}\,(2l_{x-1} + 3l_x - 6l_{x+1} + l_{x+2}), \tag{58}$$

and
$$\mu_x = \frac{1}{6l_x}\,(2d_{x-1} + 5d_x - d_{x+1}). \tag{59}$$

Here to find the respective approximate value of μ_x, terms $\frac{1}{4l_x}\Delta^4 l_x$ and $\frac{-1}{11l_x}\Delta^4 l_{x-2}$ are very small constants $\frac{6}{l_x}$ and $\frac{-2}{l_x}$ respectively when l_x is very large. Hence, it is obvious that the formulas (56), (57), (58) and (69) are exactly the formulas (31), (32), (33) and (34) respectively.

NUMERICAL EXAMPLES

Example 1. Let $l_x = 20{,}000 - 100x - x^2$ be defined at the interval $0 < x \leq 100$, find the value of the force of mortality μ_x at age 50.

Let x be the age of 48, 49, 50, 31, 52, 53 and 54, the following values can be obtained by the given function of l_x and d_x from (6):

x age	l_x	d_x
48	12,896	197
49	12,699	199
50	12,500	201
51	12,299	203
52	12,096	205
53	11,891	207
54	11,684	

From (23), we obtain

$$\mu_{50} = \frac{1}{2l_{50}}(3d_{50} - d_{51})$$
$$= \frac{1}{2.12{,}400}(3.201 - 203)$$
$$= \frac{400}{25{,}000}$$
$$= 0.016.$$

The formulas (32), (34), (43), (45) and (47) can also be used to obtain the value of $\mu_{50}=0.016$. Hence, it is evident that if l_x is a polynomial of the second degree, we can then use these formulas to find the value of μ_x.

Example 2. Let $l_x=2{,}000{,}000 - 5{,}000x - 50x^2 - x^3$ be defined at the interval $0 \leq x \leq 100$, find the value of the mortality μ_x at age 50.

Let x be the age of 48, 49, 50, 51, 52, 53 and 54, the following values can be obtained by the given function of l_x and d_x from (6):

x age	l_x	d_x
48	1,534,208	16,907
49	1,517,301	17,301
50	1,500,000	17,701
51	1,482,299	18,107
52	1,464,192	18,519
53	1,445,673	18,937
54	1,426.736	

From (32), we obtain

$$\mu_{50}=\frac{1}{6l_{50}}(11d_{50} - 7d_{51}+2d_{52})$$

$$=\frac{1}{6.1{,}500{,}000}(11.17{,}701 - 7.18{,}107+2{,}18{,}519)$$

$$=\frac{105{,}000}{9{,}000{,}000}$$

$$=0.0116666.$$

and from (34),

$$\mu_{50}=\frac{1}{6l_{50}}(2d_{49}+5d_{50} - d_{51})$$

$$=\frac{1}{6.1{,}500{,}000}(2.17{,}301+5.17{,}701 - 18{,}107)$$

$$=\frac{105{,}000}{9{,}000{,}000}$$

$$=0.0116666.$$

The formulas (43), (45) can also be used to obtain the value of $\mu_{50}=$ 0.0116666. Hence, it is evident that if l_x is a polynomial od the third degree, we can then use these formulas to find the value of μ_x.

By using the formula (23) or (51), the approximate value of μ_{50} can be obtained

$$\mu_{50}=\frac{1}{2l_{50}}(3d_{50}-d_{51})$$

$$=\frac{1}{2.1,500,000}(3.17,701-18,107)$$

$$=\frac{34,996}{3,000,000}$$

$$=0.0116653.$$

Hence, it is evident that this formula can be used to find the approximate value of μ_{50} which is very close to the exact value of $\mu_{50}=0.0116666$.

Example 3. From the Australian life table (males) 1961 (2, P. 175), the following values are reproduced:

x age	l_x	d_x	μ_x
48	89,705	587	0.00623
49	89,118	645	0.00690
50	88,473	711	0.00765
51	87,762	783	0.00850
52	86,979	860	0.00943
53	86,119	944	0.01046
54	85,175		

Find the value of μ_{50} as accurately as possible.

By using the formula (3) or (8) or (47), we obtain

$$\mu_{50}=\frac{1}{12l_{50}}[7(_{49}+d_{50})-(d_{48}+d_{51})]$$

$$=\frac{1}{12.88,473}[7(645+711)-(587+783)]$$

$$= \frac{8,122}{1,061,676}$$

$$= 0.0076501$$

which is equivalent to the value of $\mu_{50}=0.00765$, as is indicated in the Australian life table above. This formula is frequently emplyed by compilers for the constraction of a life table

By using other formulas, the approximate values of μ_{50} obtained are very close to 0.00765. We apply the formulas (43), (45), (57) and (59) and the approximate values of μ_{50} obtained will be follows:

$$\mu_{50}= \frac{1}{12l_{50}}(25_{50} - 23d_{51}+13d_{52} - 3d_{53})=0.0076426,$$

$$\mu_{50}= \frac{1}{12l_{50}}(3d_{49}+13d_{50} - 5d_{51}+2d_{52})=0.0076511,$$

$$\mu_{50}= \frac{1}{6l_{50}}(11d_{50} - 7d_{51}+2d_{52})=0.0076482,$$

and $$\mu_{50}= \frac{1}{6l_{50}}(2d_{49}+5d_{50} - d_{51})=0.007652.$$

Example 4. Let $l_x=10,000(100 - x)^{1/2}$ be defined at the interval $0 \leq x \leq 100$, find the approximate value of μ_{50}.

Let x be the age of 48, 49, 50, 51, 52, 53 and 54, the following values can be obtained by the given function of l_x and d_x from (6):

x age	l_x	d_x
48	72,111	697
49	71,414	704
50	70,710	710
51	70,000	718
52	69,282	726
53	68,556	734
54	67,822	

By using the formula (3), (8) or (47) as it is frequently used by compilers for the construction of a life table, we obtain

$$\mu_{50}=\frac{1}{12l_{50}}[7(d_{49}+d_{50})-(d_{48}+d_{51})]$$

$$=\frac{1}{12.70,710}[7(704+710)-(697+718)]$$

$$=\frac{8,483}{848,520}$$

$$=0.0099974.$$

By using other formulas, the approximate values of μ_{50} obtained are very close to 0.0099974. For instance, with the formulas (43), (45), (57) and (59), we shall have the approximate values of μ_{50} as follows:

$$\mu_{50}=\frac{1}{12l_{50}}(25d_{50}-23d_{51}+13d_{52}-3d_{53})=0.0099844,$$

$$\mu_{50}=\frac{1}{12l_{50}}(3d_{49}+13d_{50}-5d_{51}+2d_{52})=0.0099915,$$

$$\mu_{50}=\frac{1}{6l_{50}}(11d_{50}-7d_{51}+2d_{52})=0.0099844,$$

and

$$\mu_{50}=\frac{1}{6l_{50}}(2d_{49}+5d_{50}-d_{51})=0.0099938.$$

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